# Early bearing defect detection in a noisy environment based on a method combining singular value decomposition and empirical mode decomposition

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# Abstract

This paper proposes a new method combining Empirical Mode Decomposition (EMD) and Singular Value Decomposition (SVD) for bearing fault diagnosis. The method includes three steps. First, the signal is decomposed using EMD. Secondly, the instantaneous amplitudes are computed for each component using the Hilbert Transform (HT). Lastly, the Singular Value Vector is applied to the matrix of Cross-Power Spectral Density (CPSD) of the instantaneous amplitude matrix and the SVD versus frequency is analysed. The proposed method is first validated by using various noisy simulated signals. The results show that the proposed method is robust versus the noise to detect the bearing frequencies that are representative of the defect even in a very noisy environment and that the amplitude of the first SVD at each bearing frequency is very sensitive to the defect severity. The proposed method is also applied to two different experimental cases on bearings with very low degradation. The results show that the proposed method is also applied to two different experimental cases on bearings with very low degradation for both experimental cases.

**Keywords:** Bearing fault, Empirical Mode Decomposition (EMD), Hilbert transform (HT), Cross-Power Spectral Density (CPSD), Singular Value Decomposition (SVD).

# 1. Introduction

Bearing wear can be considered as a major cause of defects in rotating machinery. Unexpected failures in bearings may cause significant economic losses. Empirical Mode Decomposition (EMD) is an interesting technique for fault diagnosis of rotating machinery. EMD can decompose the signal into several components called Intrinsic Mode Functions (IMFs) [1]. With EMD, the principal "modes" representing the signal can be identified. This method has attracted much attention for signal processing and engineering applications over the past decade [2]. The fundamental idea when using the EMD method is to decompose the vibratory signal into multiple components and the suitable IMF allows for computing the envelope spectrum and analyse their statistical features. Hybrid methods based on EMD and other techniques, like the Wavelet Packet Transform (WPT), the Support Vector Machine (SVM), Spectral Kurtosis (SK) and the Teager-Kaiser Energy Operator (TKEO), have also been applied to bearing fault diagnosis [3–7].

After performing EMD on a signal, some IMFs are associated to bearing faults, others with information unusable for diagnosing such faults. The useful IMF (if it exists) can be selected to perform the Hilbert spectrum. A few studies focus on developing an indicator to select automatically this useful IMF. Wenliao *et al.*[8] used the Wigner-Ville distribution to select the optimum IMFs and the filter bandwidth. Ricci *et al* [9] proposed a new indicator, named Merit Index, to select the appropriate IMF. The Merit Index is a linear combination between the periodicity degree of the IMF and its absolute skewness value. Yi *et al.*[10]

proposed a new indicator called Confidence Index based on combination of correlation coefficient, skewness and kurtosis. Jacek *et al.*[11], Peng et al. [12], Wang *et al.* [13], and Guo and Tse [14] utilized the correlation coefficient as an indicator analysis to select the candidate IMFs.

However, it is well known that the repetitive impacts due to the defect excite all bearing natural frequencies. If only the best IMF is selected, information included in other IMFs excited by the fault is then lost. Selecting all IMFs excited by the fault appears thus more suitable. In [15-16], the authors propose to select all the IMFs excited by the fault. A hybrid method based on EMD and run-up excitation is proposed to select the useful IMFs. By using a swept excitation when running up a rotating machine, the resonance frequency bands of the mechanical system is obtained from the spectrogram of the signal. In [17-18], the authors proposed to select all IMFs selected by the fault for early detection of the defect. The selection is made through an indicator-based kurtosis.

In this study, a new approach exploiting all IMFs of the signal to improve fault diagnosis is proposed. To compress all information extracted from each IMF, Singular Value Decomposition (SVD) is used in this paper. The SVD method has been widely used in fault feature extraction and identification for mechanical systems [19–21]. Before performing SVD, a matrix obtained from the original one-dimensional signal must be constructed. Various matrices exist, for example: the Toeplitz matrix, cycle matrix and Hankel matrix. To improve fault feature extraction, a new approach exploiting the instantaneous amplitude of each IMF obtained by EMD is proposed. The matrix is defined as the Cross-Power Spectral Density (CPSD) of all instantaneous amplitudes of the obtained IMF. Cross-spectral analysis is a powerful tool for investigating the relationship between signals in the frequency domain. Inspired by the frequency domain technique [17], the power spectral density matrix for each frequency is decomposed by applying SVD to the matrix. The singular value plot of the spectral density matrix concentrates information from all spectral density functions. The first singular value should approximately equal the sum of the terms on the diagonal of the PSD matrix. This means that the power of the signals at this frequency can be attributed to the vibratory signature [22]. The following sections give details of the proposed method used for rolling bearings fault diagnosis. The paper first presents the theoretical background of EMD and the proposed approach in Section 2. To validate the approach, the method is applied to a simulated signal and real data from damaged bearing in sections 3 and 4. Section 5 concludes the paper.

### 2. The proposed approach

#### 2.1 A brief description of EMD

The EMD method can decompose signal in a multiple intrinsic mode functions (IMFs). The decomposed signal may be written as [1-2]:

$$x(t) = \sum_{i=1}^{N} C_i(t) + r_N(t)$$
(1)

where  $C_i(t)$  is the *ith IMF* and  $r_N(t)$  is the residual signal.

This method could suffer of a mixing mode problem and derived methods such as EEMD, CEEMD, CEEMDAN could also be used in this case, but it is not always necessary.

#### 2.2 The proposed approach based on SVD and EMD

The approach proposed for diagnosing faults on rolling bearings is illustrated in Figure 1. In this paper, the signal matrix obtained from EMD is constructed as follows:

 $[C_1; C_2; C_3; ... C_n]$ 

(2)

The first step is to compute the instantaneous amplitude of each intrinsic mode function  $C_i(t)$ . The instantaneous amplitude (*IA*) is computed by means of the Hilbert Transform. The analytical signal is given by the following expression:

$$z(t) = C(t) + j\tilde{C}(t) = a(t)e^{i\varphi(t)}$$
(3)

where a(t) is the amplitude function given by the expression:

$$IA_{i}(t) = a(t) = \sqrt{\left(C_{i}\right)^{2} + \left(\tilde{C}_{i}\right)^{2}}$$
(4)

The signal matrix of instantaneous amplitudes is constructed as follows:

$$M = [IA_1; IA_2; IA_3; ... IA_n]$$
(5)

The second step is to compute the cross-power spectral density of matrix M. The cross-power spectral density is defined by [23]:

$$P_{xy}(w) = \sum_{m=-\infty}^{\infty} R_{xy}(\tau) \ e^{-jw\tau}$$
(6)

where  $R_{xy}(\tau) = E\{x(t)y(t+\tau)\}$  with  $E[\cdot]$  denoting the expectation operator on t. (7)

The diagonal elements of the matrix represent the auto-power spectral density (the same IA). The offdiagonal elements are the complex cross-spectral densities between two different IA.

$$P(w_{i}) = \begin{bmatrix} P_{IA_{1}IA_{1}}(w_{i}) & P_{IA_{1}IA_{2}}(w_{i}) & \dots & P_{IA_{1}IA_{n}}(w_{i}) \\ P_{IA_{2}IA_{1}}(w_{i}) & P_{IA_{2}IA_{2}}(w_{i}) & \dots & P_{IA_{2}IA_{n}}(w_{i}) \\ \dots & \dots & \dots & \dots \\ P_{IA_{n}IA_{1}}(w_{i}) & P_{IA_{n}IA_{2}}(w_{i}) & \dots & P_{IA_{n}IA_{n}}(w_{i}) \end{bmatrix}$$

$$(8)$$

The flow chart of the proposed method is shown in Fig. 1.



Figure 1: Flow chart of the proposed method

Assuming that  $P(w_i)$  is an  $n \times n$  matrix, the power spectral density matrix for each frequency  $(w_i)$  is decomposed by applying SVD to matrix  $P(w_i)$ .

$$P(w_i) = U \sum V^T$$
(9)  
where U and V are orthogonal and  $\Sigma$  is a diagonal matrix of the singular values  
 $(\sigma_{1,1} \ge \sigma_{2,2} \ge \sigma_{3,3} \dots \ge 0 \text{ and } \sigma_{k,j} = 0 \text{ if } k \ne j ).$ 

$$\Sigma(w_i) = \begin{bmatrix} \sigma(w_i)_{1,1} & \dots & \dots & 0 \\ 0 & \sigma(w_i)_{2,2} & \dots & 0 \\ 0 & \dots & \dots & 0 \\ 0 & \dots & \dots & \sigma(w_i)_{n,n} \end{bmatrix}$$
(10)

As mentioned in the introduction, the first singular value should approximately equal the sum of the terms on the diagonal of the PSD matrix [22]. The plot of the first singular value versus frequency is thus used to identify the features extracted from the signal.

# 3. Validation with simulated data

#### **3.1 Fault detection**

In order to validate the proposed method and evaluate its effectiveness, a simulated numerical bearing signal is used. The simulated signal is similar to the signal used in [17-18]. The mathematical expression of the signal is given as:

$$x(t) = Ae^{-\alpha t'} \sin\left(2\pi f_n t\right) + n(t) \tag{11}$$

where

$$t' = mod\left(t, \frac{1}{F_m}\right) \tag{12}$$

Resonant frequency  $f_n$  is set to 1,800 Hz. The BPFO is set to 100 Hz. Amplitude A is set to 1. Sampling frequency  $F_s$  is set to 12,000 Hz. A random signal n(t) with variance  $\sigma^2 = 0.01$  is added to x(t).

Figure 2 illustrates the simulated roller bearing signal. Figure 3 shows the 12-IMF obtained by the EMD method. It can be seen from this figure that the shocks related to defect are distributed across the first eight IMFs. As discussed in Section 2, all IMFs are considered in the proposed approach. The result is presented in Figure 4. Figure 4 exhibits the plot of the first singular value versus frequency. The fundamental of the BPFO (100 Hz) and its harmonics up to 1,200 Hz are clearly identified. The initial conclusion is thus that the proposed method can effectively detect the defect. No need to select the useful IMFs to accomplish the diagnosis.



Figure 2: Simulated signal



Figure 3: EMD of the simulated signal



Figure 4: Result obtained for the simulated signal

It is well know that the amplitude of vibration due to bearing defects increases as the fault worsens and high peak levels may be observed. To confirm the efficiency of the method and its sensitivity to the severity of defect-induced vibrations, the simulated signal given by Equation (11) is simulated with A set to 1, 1.3, 1.6 and 1.9. The resulting waveforms are given in Figure 5.



Figure 5: Simulated signal for different levels of A

Figure 6 shows the results obtained by the new approach for different levels of A. The magnitude of the BPFO and number of bearing frequency harmonics increase as the value of A increases. As seen in Figure 7, when A = 1.3, the magnitude of the BPFO increases by 6 dB. The same conclusions may be drawn from the cases shown in Figure 7A. Therefore, the method is sensitive to the severity of the shocks. BPFO magnitude and the mean of all harmonics can be used as an indicator to track the severity of the defect.



Figure 6: Results obtained for different levels of A



Figure 7: (A) Change in BPFO (B) Change in mean amplitude of all harmonics in Figure 6

The noise level in the simulated signal above is fairly low, though any industrial application would probably involve additive noises, potentially masking the signature of the defect, especially in the case of early bearing degradation. Another test was thus conducted to prove that the proposed method is able to detect the defect even if the noise level is higher.

#### **3.2** Sensitivity to noise

The effectiveness of this method is evaluated based on its ability to extract bearing fault-related information. When a bearing is at an early stage of degradation, the signature of the defect may be masked by noise and difficult to extract. Gaussian white noise signals with variance  $\sigma^2$  of 0.05, 0.1, 0.2 and 0.4 were thus added to the original simulated signal, x(t), in order to evaluate the ability of the method to extract the defect-related information or signature when it is completely hidden by noise.

Figure 8 shows the simulated signals with the four values of variance  $\sigma^2$  of added noise. It can be seen that defect-related shocks are masked by noise for  $\sigma^2 = 0.1$ , 0.2 and 0.4. The four noisy signals were processed using the proposed method and the results obtained from all signals are exhibited in the Figure 9. Note that even when the noise is very high, the method is able to identify the BPFO, while other tested state of the art methods were not available at this high noise level.



Figure 8: Simulated noisy signals: (A)  $\sigma^2 = 0.05$ ; (B)  $\sigma^2 = 0.1$ ; (C)  $\sigma^2 = 0.2$ ; (D)  $\sigma^2 = 0.4$ 



Figure 9: First singular value in frequency domain: (A)  $\sigma^2 = 0.05$ ; (B)  $\sigma^2 = 0.1$ ; (C)  $\sigma^2 = 0.2$ ; (D)  $\sigma^2 = 0.4$ 

# 4. Experimental data

The proposed method was investigated on two different history cases with different low levels of severity.

# 4.1 First case study

In the first case study, two defected bearings (SKF 1210 EKTN9) with very low severity levels are investigated. The test bench is shown in (Figure 10A). The first defected bearing noted D1, has an artificial defects about 200  $\mu$ m deep with a groove width (*W*) of 50  $\mu$ m and the second (D2) with *W* = 100  $\mu$ m. The data were recorded when rotating at 600 rpm, using an accelerometer with a sensitivity of 100 mV/g. The frequency of the BPFO is about 76.46 Hz. The sample frequency is set to 12500 and 64,000 samples are used (acquisition time of 5 seconds).



Figure 10: Test bench

Figure 11 shows the time signal of the acquired data. For the healthy bearing (Figure 11-A), quasirandom shock signals are observed. Defect D1 (50  $\mu$ m) is at early stage of degradation. The time signal of defect D2 (Figure 11C) shows a series of impulse responses at BPFO and the amplitude is modulated periodically at the shaft speed. The shocks caused by defect D2 are more perceptible in the D2 signal than those due to defect D1, which are masked by noise.



Figure 11: (A) Healthy bearing, (B) Defective bearing D1, (C) Defective bearing D2

The signals were processed using the proposed method. The defective bearing (50  $\mu$ m) very clearly shows the BPFO (72.46 Hz) and its harmonics (Fig. 12). For defect D2 (100  $\mu$ m), the BPFO harmonics are clearly identified and an increase in amplitude is observed (Fig. 13). The amplitude of the BPFO increases by 12.43 dB. This reveals that defects are more clearly identified and well-defined using the proposed approach.



Figure 12: First singular value in frequency domain: Defect D1 (red), healthy bearing (blue)



Figure 13: First singular value in frequency domain: Defect D2 (red), healthy bearing (blue)

#### 4.2 Second case study

In this case, bearing (SKF 6205-2RS JEM ) with defects in different locations are investigated. The vibration data are available in [24]. The test bench is shown in Figure 14. The first defect has a simulated single-point fault on inner race and the second defect has a simulated single-point fault on outer race. The fault size is 0.007" in diameter and 0.011" in depth.. The vibration data was collected when rotating at 1796 rpm. The frequency of the BPFO is about 107.6 Hz and the BPFI is 161.4 Hz. The sampling frequency (*Fs*) is 12,000 Hz and 60,000 data samples are used.



Figure 14: Test bench



Figure 15: (A) Healthy bearing, (B) Inner ring fault (C) Outer ring fault

The time signals of the healthy bearing, inner ring fault and outer ring fault are shown in Figure 15. The time signal of the inner ring fault in Figure 15B shows that the amplitude is modulated periodically at the shaft speed. This is due to the rate at which the fault passes through the load zone. The signal is processed with the proposed method. Figure 16 illustrates the result. The result shows a series of harmonics of BPFI at 161.4 Hz, with sidebands spaced at the shaft speed to either side of each harmonic, as well as a number of harmonics of the shaft speed. This phenomena is usually related to a bearing looseness [25]. The

time signal of outer ring fault is shown in Figure 15C. Unlike that of the inner ring defect, the time signal of the outer ring fault should reveal a series of uniform impulse, but the signal is modulated at the shaft speed. The result obtained by the proposed method is shown in Figure 17. A series of harmonics of the BPFO are detected. As explained in [25], this modulation suggests a rotating load caused probably by mechanical looseness.



Figure 16: First singular value in frequency domain: Inner race fault (red), healthy bearing (blue)



Figure 17: First singular value in frequency domain: Outer race fault (red), healthy bearing (blue)

# 5 Conclusion

A new feature extraction method for bearing fault diagnosis is presented in this paper. In the proposed method, there is no need to select the useful IMF to accomplish the diagnosis. The matrix of cross-power spectral density of all IMFs is decomposed in the frequency domain using SVD to extract defect-related information. The method was first validated by means of a simulated signal. The results have shown that this method may be used even in a very noisy environment. The proposed method can effectively detect defects even if the induced shocks are completely masked by noise and that the features extracted are sensitive to defect shock amplitude, making them useful indicators to track defect severity. Two test cases are presented to verify the efficiency of the method. Bearings in an early stage of degradation with two levels of defect severity (50  $\mu$ m and 100  $\mu$ m) and defects in different locations are investigated. The results show that the method can effectively extract all information related to the defect. In this study, we validated the proposed method using rolling bearings. Future work will extend and generalize the method for fault diagnosis of other rotating machinery, such as gears.

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