ARX model for experimental vibration analysis of flexible manipulator during grinding

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Abstract

Using a flexible manipulator for grinding process in situ has become a cost effective engineering service in the recent years, especially for repair and refurbish of mechanical systems and components. In comparison with traditional rigid robot manipulators, the flexible manipulator has proved its efficiency in terms of accuracy and facility. However, because of its compact and flexible structure, concerns arise regarding its dynamic behavior during a grinding process. This paper proposes a method using an ARX (autoregressive with exogenous excitation) model for experimentally analyzing the vibrations of a flexible robot during a grinding operation in different cases: Single Input–Single Output (SISO) and Multi Input–Multi Output (MIMO). Simultaneously, a dynamometer allows for triaxial input force measurement while three accelerometers mounted at the end effector record the vibration outputs. Due to the Operational Modal Analysis (OMA), the dynamical properties of the robot can be identified directly during operation. The results have shown that the ARX model is efficient for analyzing the operational vibration in complex systems with multi degrees of freedom and multi directions. The determination of modal parameters and identified Frequency Response Functions (FRFs) enable to predict the dynamical behavior of the system and to simulate the vibration in real working conditions. Further studies on inverse problem are promising for estimating the excitation forces while these later are not available and not practically measured in industrial applications.

Keywords: Operational modal analysis, flexible manipulator, grinding process, ARX model, transfer functions, force identification.

1. Introduction

Nowadays robots sufficiently conduct manifold manipulation works with a high degree of autonomy and rigorousness. Portable manipulator systems are regarded as an effective and profitable solution for the automation maintenance tasks on large hydroelectric equipment. The SCOMPI (Super COMPact robot Ireq) was developed at IREQ (Hydro Quebec's research institute) and is particularly designed with flexible links and flexible joints for working in the hard-to-reach areas or confined spaces of hydraulic turbines in a hostile environment [1]. Because of its flexible structure, vibration problems of Scompi become crucial since producing chatter and bad surface finish. A numerical simulation [2] has been constructed in MSC/Adams in different configurations included impact force, sinusoidal and operational forces. There is a great number of researches that focus on identifying the modal parameters of the system in order to understand the dynamical behavior of robot [3-7], and estimate the operational forces from the actual accelerations measured on the robot [8]. Knowing a system's frequency response function is a key to many system analysis and control synthesis methods [9]. The main problems are due to the fact that these modal parameters are changing with the robot motion and position and thus a time-varying method is proposed for studying this kind of non-stationary structure [10, 11]. Researchers are particularly interesting to identification of continuous-time system by using discrete data [12].

This paper presents a technique to identify the modal parameters as well as the transfer function of Scompi robot by applying the Autoregressive with eXogenous input (ARX) model [12-14]. This method reveals a convenient and advantageous for Operational Modal Analysis of structures (OMA), which allows for determining operational modal model excited by ambient noise and vibration. The modal parameters are estimated and identified by applying straightforward method such as Ordinary Least Squares (OLS) [12], [15]. The results are validated by another approach based on updated Auto Regressive (AR) model in [4] and shown a great accuracy of identified modal parameters. This study enables us to predict the dynamical behavior of Scompi for identifying excitation forces during operations of grinding and consequently improve the quality of the surface finish.

2. Auto Regressive Exogenous Excitation model (ARX)

The ARX model [12-15] is a primary choice because of its simplicity. It has been applied to numerous practical applications especially in control systems. However, critical motivation for choosing the ARX model, is its correlation to the state space model [16-19] which can be implemented for inverse problem with the aim of reconstructing the excitation forces acting on vibrating structures [8], which is impossible to obtain from direct measurement in the real systems. The ARX model is a convenient model to obtain the general relation between input and output signals for different cases, such as Single Input – Single Output (SISO) or Multiple Input – Multiple Output (MIMO), which can reliably represent the dynamic properties of the system. Figure 1 illustrates the block diagram of ARX model.

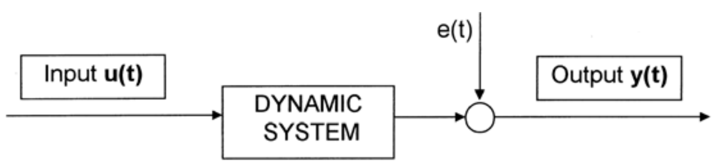

Figure 1. Block diagram of ARX model

This model has a simple structure and strong robustness. It is very efficient when the noise is low. However, when the noise is large, the order of the model must increase to compensate the impact to system identification precision from noise [18].

Examine a *c* dimensional vector input **u**(t) and a *d* dimensional vector output **y**(t) of a Multiple Input and Multiple Output (MIMO) system.

The ARX model can be described as a linear difference equation:

$$\mathbf{y}(t)+\mathbf{A}_1\mathbf{y}(t-1)+\dots+\mathbf{A}_{n_a}\mathbf{y}(t-n_a)=\mathbf{B}_0\mathbf{u}(t)+\mathbf{B}_1\mathbf{u}(t-1)+\dots+\mathbf{B}_{n_b}\mathbf{u}(t-n_b)+\mathbf{e}(t) \tag{1}$$

where:

$\mathbf{A}_i$ – are *d x d* matrices and

$\mathbf{B}_i$ – are *d x c* matrices.

The general ARX model can be rewritten in the polynomial form:

$$\mathbf{A}(q)\,\mathbf{y}(t)=\mathbf{B}(q)\,\mathbf{u}(t)\ +\mathbf{e}(t) \tag{2}$$

where:

$$\mathbf{A}(q)=\mathbf{I}\ +\mathbf{A}_1 q^{-1}+\mathbf{A}_2 q^{-2}+\ \ldots+\mathbf{A}_{n_a} q^{-n_a} \tag{3}$$

$$\mathbf{B}(q)=\mathbf{B}_0+\ \mathbf{B}_1 q^{-1}+\mathbf{B}_2 q^{-2}+\ldots+\mathbf{B}_{n_b} q^{-n_b} \tag{4}$$

The model (2) is an ARX model where AR refer to the Autoregressive part **A**(q)**y**(t) and X refer to the extra input **B**(q)**u**(t) called the exogenous input. **y**(t) is considered as the output of the model while **u**(t) is the input to the model and **e**(t) is innovation term at the time *t*. **A**(q) and **B**(q) are polynomials in the delay operator q^{-1} and n_a, n_b are the model order of **A**(q) and **B**(q) respectively. **A**(q) is a matrix whose elements are polynomials in q^{-1}.

This results in Matrix Fraction Description (MFD).

Defining the parameter matrix:

$$\theta=[\mathbf{A}_1\ \ \mathbf{A}_2\ \ \ldots\ \ \mathbf{A}_{n_a}\ \ \mathbf{B}_0\ \ \mathbf{B}_1\ \ \ldots\ \ \mathbf{B}_{n_b}\ ]^T \tag{5}$$

We may rewrite (2) as a linear regression:

$$y(t)=\theta^T\phi(t)+e(t) \tag{6}$$

If we consider N consecutive values of the responses from *y(k)* to *y(k+N-1)*, the model parameters can be obviously estimated by least square method [15] by minimizing the norm of *e(t):*

$$\Phi=\arg\min\left(\frac{1}{N}\sum_{t=k}^{k+N-1}\left\|e(t)\right\|^2\right)=\arg\min\left(\frac{1}{N}\sum_{t=k}^{k+N-1}\left\|\mathrm{y}(t)-\theta^T\phi(t)\right\|^2\right) \tag{7}$$

After obtaining the measured force and acceleration signals on all channels, the model ARX can be used to fit the data. The ARX model creates a regressive connection between the input vector **u**(t) and the output vector **y**(t) through a residual vector **e**(t). By applying the least square method, the modal parameters matrices **A** and **B** can be estimated. In vibration measurement application, it can be seen that force (input) and acceleration (output) are normally synchronized, thus the two parts may be modeled with the same order $n_a = n_b$.

Once the model parameters of the system are identified, the state matrix can be determined as in the form of autoregressive parameters:

$$A\Big|_{(naxnb)}=\begin{bmatrix}-A_1 & -A_2 & -A_3 & \ldots & -A_p\\ I & 0 & 0 & \ldots & 0\\ 0 & I & 0 & \ldots & 0\\ \ldots & \ldots & \ldots & \ldots & \ldots\\ 0 & 0 & 0 & I & 0\end{bmatrix} \tag{8}$$

There is a remarkable coincidence that the poles of model are also the roots of characteristic polynomial of the state matrix. Consequently, the continuous eigenvalues, system natural frequencies and damping rates of the structure can be calculated for each pole by using the subsequent standard equations:

Eigenvalues: $$[V,\lambda]=eig\,(A) \tag{9}$$

Frequencies: $$f_i = \frac{\sqrt{\mathrm{Re}^2(\lambda_i) + \mathrm{Im}^2(\lambda_i)}}{2\pi} \tag{10}$$

Damping rates: $$\xi_i = -\frac{\mathrm{Re}(\lambda_i)}{2\pi f_i} \tag{11}$$

When the modal parameters are estimated, we can construct the transfer function which is regarded as the frequency response function of the system. All the system can be described by linear constant coefficients and represented by transfer functions that are "rational polynomial in q".

$$\mathbf{G}(q) = \frac{\mathbf{B}(q)}{\mathbf{A}(q)} = q^{-n_k} \frac{\mathbf{B}_1 q^{-1} + \mathbf{B}_2 q^{-2} + \ldots + \mathbf{B}_{n_b} q^{-n_b}}{\mathbf{I} + \mathbf{A}_1 q^{-1} + \mathbf{A}_2 q^{-2} + \ldots + \mathbf{A}_{n_a} q^{-n_a}} \tag{12}$$

with n_k is the transport delay.

3. Application to a flexible manipulator during grinding process

3.1 Brief introduction of the SCOMPI robot

The proposed approach is now implemented to the portable robot Scompi. Figure 2 presents the structure of Scompi, which is used for repair tasks in Hydro Quebec power plants, particularly for grinding or welding jobs [1]. Because of its compact and flexible structure, the question is raised up from its dynamical behavior under operating conditions. Hence, the flexibility of the joints and links needs to be taken into consideration, which might affect the stabilization of robot at the end effector during operational process [4]. The aim of Scompi is to achieve both a high Material Removal Rate (MMR) and a polished surface finish with great precision. However, because of the portable and lightweight design, undesired chatter vibrations can appear during machining process which produces an undesirable waviness surface. Therefore, the monitoring of its modal parameters as well as the transfer functions of the structure in the grinding operation are necessary for minimizing vibration at the end effector while controlling chatter phenomenon and improve the quality of grinding surface.

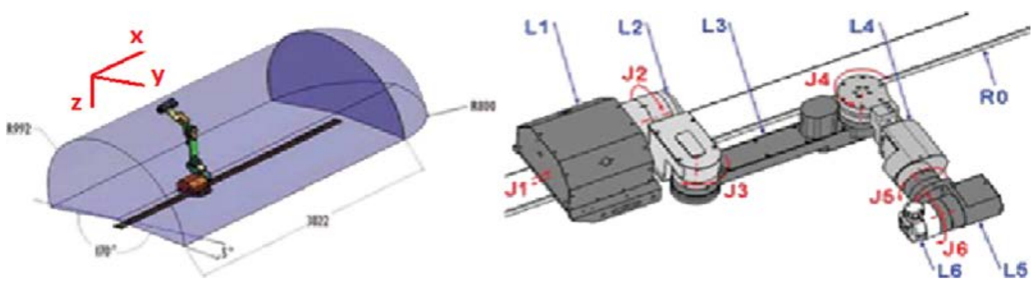

Figure 2. Scompi robot

3.2. Presentation of the Experimental setup

As can be seen from figure 3, a Scompi robot is tested under real grinding operation. Due to the interest in typical dynamic behavior of the robot at the end effector, the Scompi is set to its home configuration. Three accelerometers are mounted at the end effector in triaxial directions X, Y and Z. Meanwhile, a Kistler table dynamometer CH8408 is placed under the work-piece for measuring the forces. The power is set up at 1500 W and grinding motor is rotated at a constant speed of 3225 (rpm) for conducting each single grinding pass within 12 seconds. A multi-component dynamometer is used for measuring the grinding forces in three directions at the tool piece contact point. After obtaining the measured signals from dynamometer and accelerometers, we acquired them to the frequency rate of 512 (Hz) (Figure 4, 5).

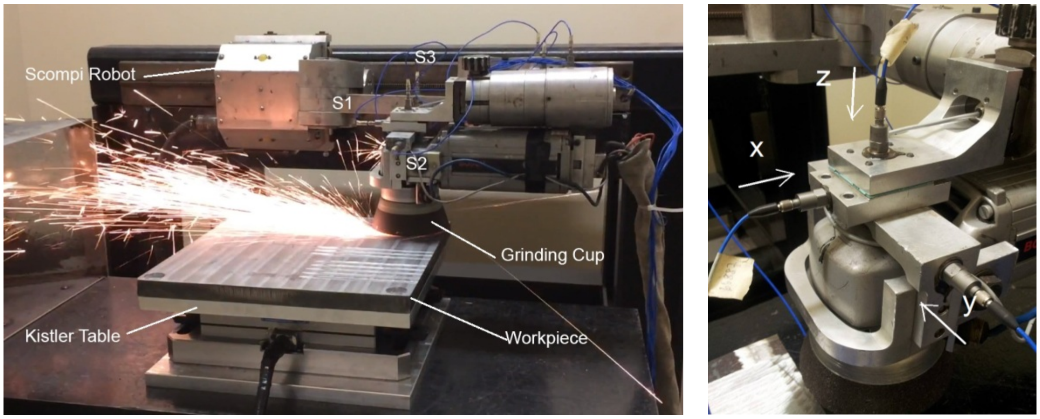

Figure 3. Overall configuration of the experimental setup

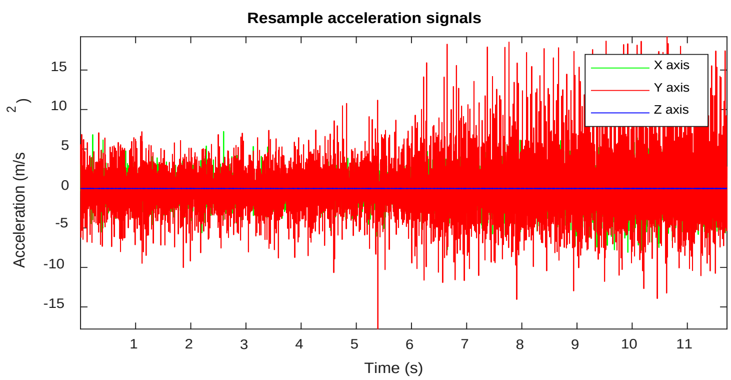

Figure 4. Measured acceleration signals during grinding process.

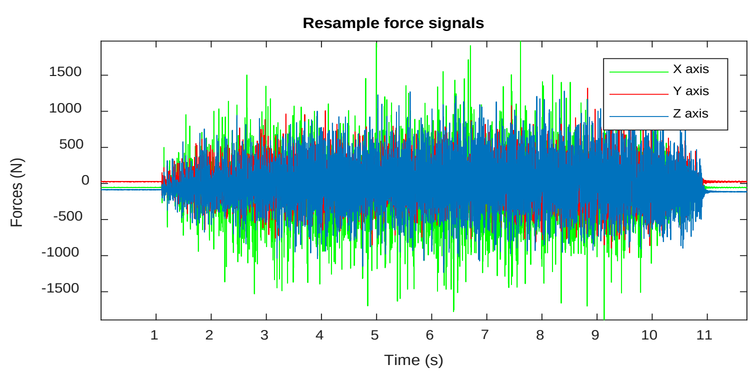

Figure 5. Measured force signals during grinding process.

Taking three measured acceleration signals in X, Y and Z directions, by applied Fast Fourier Transform (FFT) analysis, we can easily see the measured signals in both time and frequency domain as shown in Figure 6. As indicated, there are some significant frequencies in frequency domain such as 53.9 (Hz) - the first harmonic; 93.7 (Hz); 106.2 (Hz) - the second harmonic; 146.8 (Hz) and 200.8 (Hz).

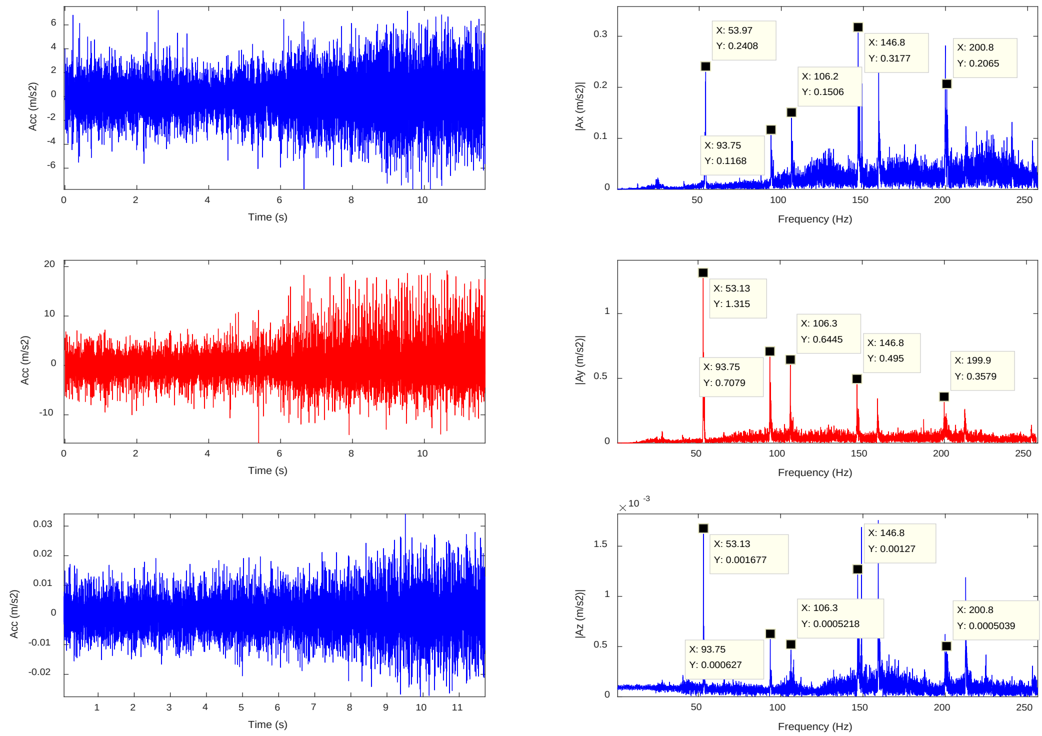

Figure 6. Time domain and frequency domain of the acceleration signals in three directions.

4. Results and discussion

Operating in a tridimensional space, the ARX model is applied on Scompi structure to fit the measured signals on each direction ($S_1 - F_x$); ($S_2 - F_y$); ($S_3 - F_z$) for constructing frequency stabilization in different cases: Single Input – Single Output (SISO) and Multi Input – Multi Output (MIMO). The figures 7–10 demonstrated the frequency stabilization diagrams up to 250 (Hz) with a model order up to 100 where all the interesting frequencies may be observed. The model order is chosen at 100 for computation of the modal parameters with low uncertainties. In addition, another stabilization given in figure 11 is computed by MODALAR based on updated AR model [4] with an aim of validation between two approaches. The 53.75 (Hz) electric frequency of grinding and its harmonics are clearly revealed in the stabilization diagrams.

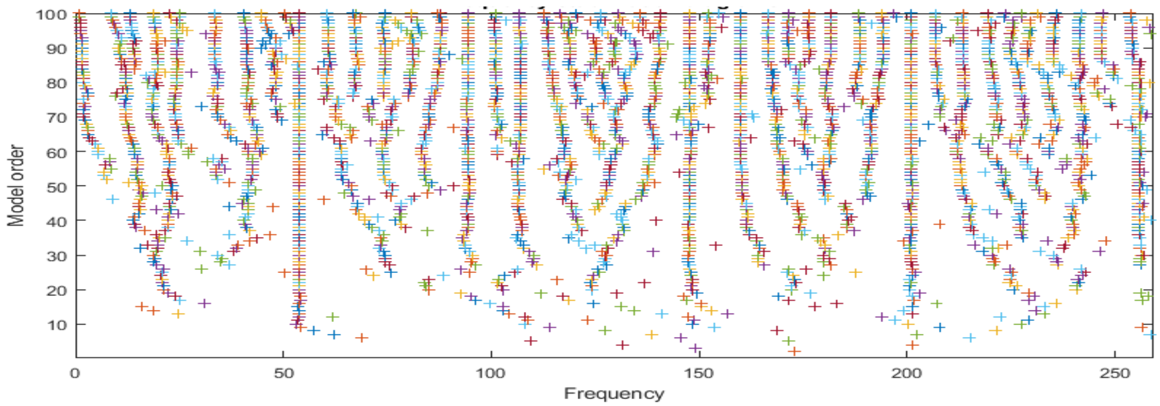

Figure 7. Frequency stabilization diagram on X direction

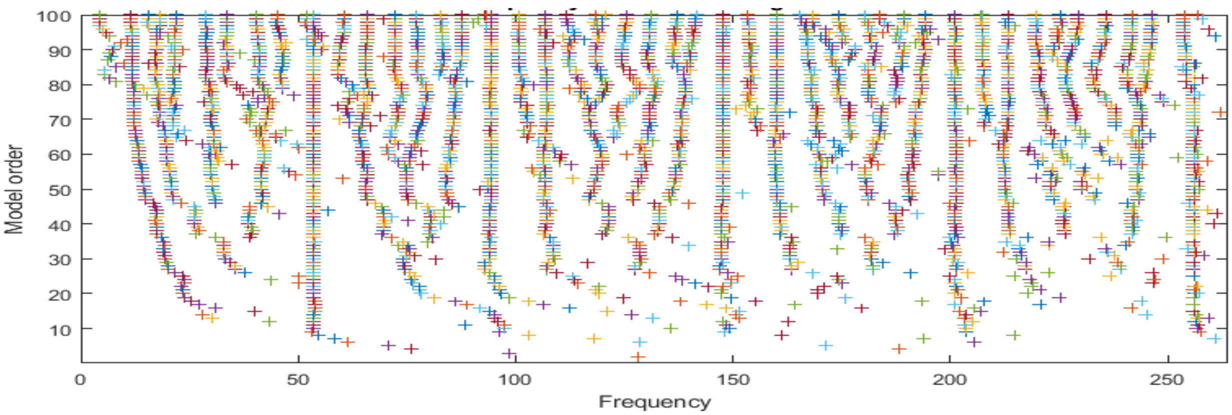

Figure 8. Frequency stabilization diagram on Y direction

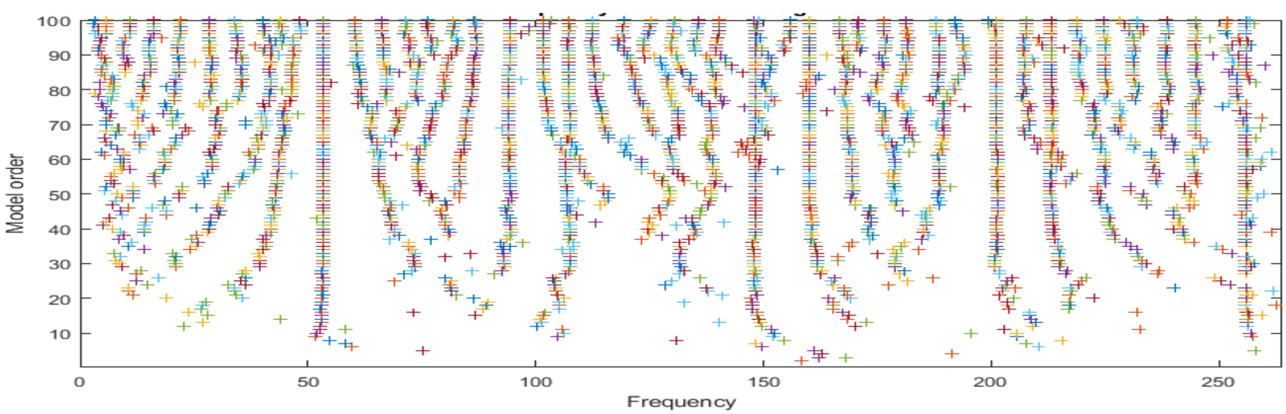

Figure 9. Frequency stabilization diagram on Z direction

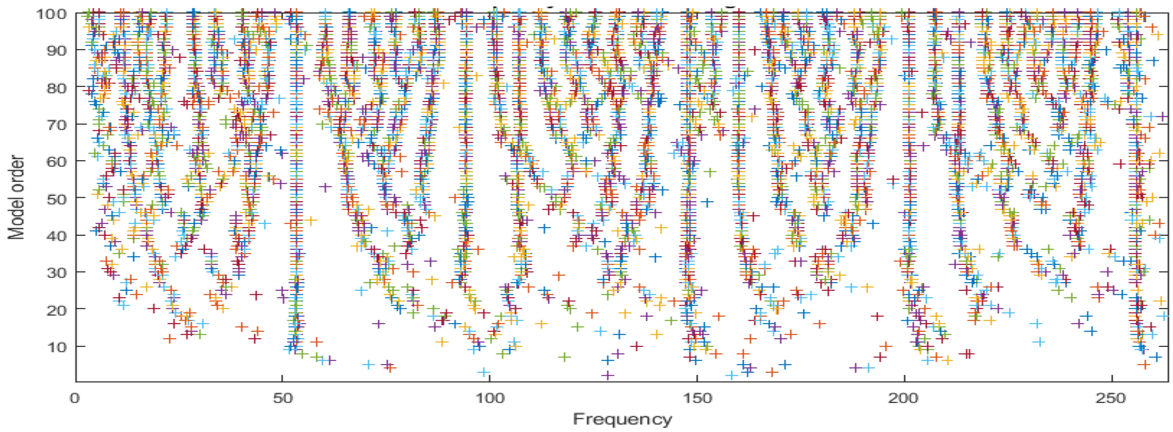

Figure 10. Frequency stabilization diagram in MIMO case (three inputs and three outputs)

Synthetically, the natural frequencies and damping ratios are estimated directly from the frequency stabilization diagram of MIMO case, where all the excited frequencies can observed clearly in multi directions. Figure 12 illustrated the stabilization diagrams of damping ratio with 95% uncertainties. The natural and harmonic frequencies identified by two methods with their damping ratio are given in the table 1. The harmonic frequencies are identified with their damping rates close to zero.

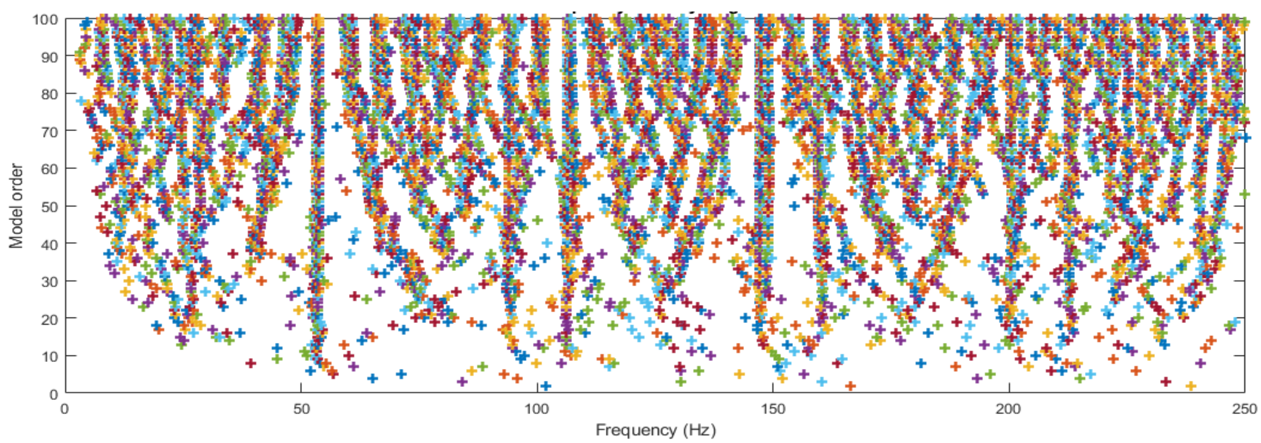

Figure 11. Frequency stabilization diagram by MODALAR

Figures 13-21 present the transfer functions identified by ARX model at the order 100. The identified transfer function from the working condition is crucial for the assessment of the robot dynamics and for further simulations under different loadings.

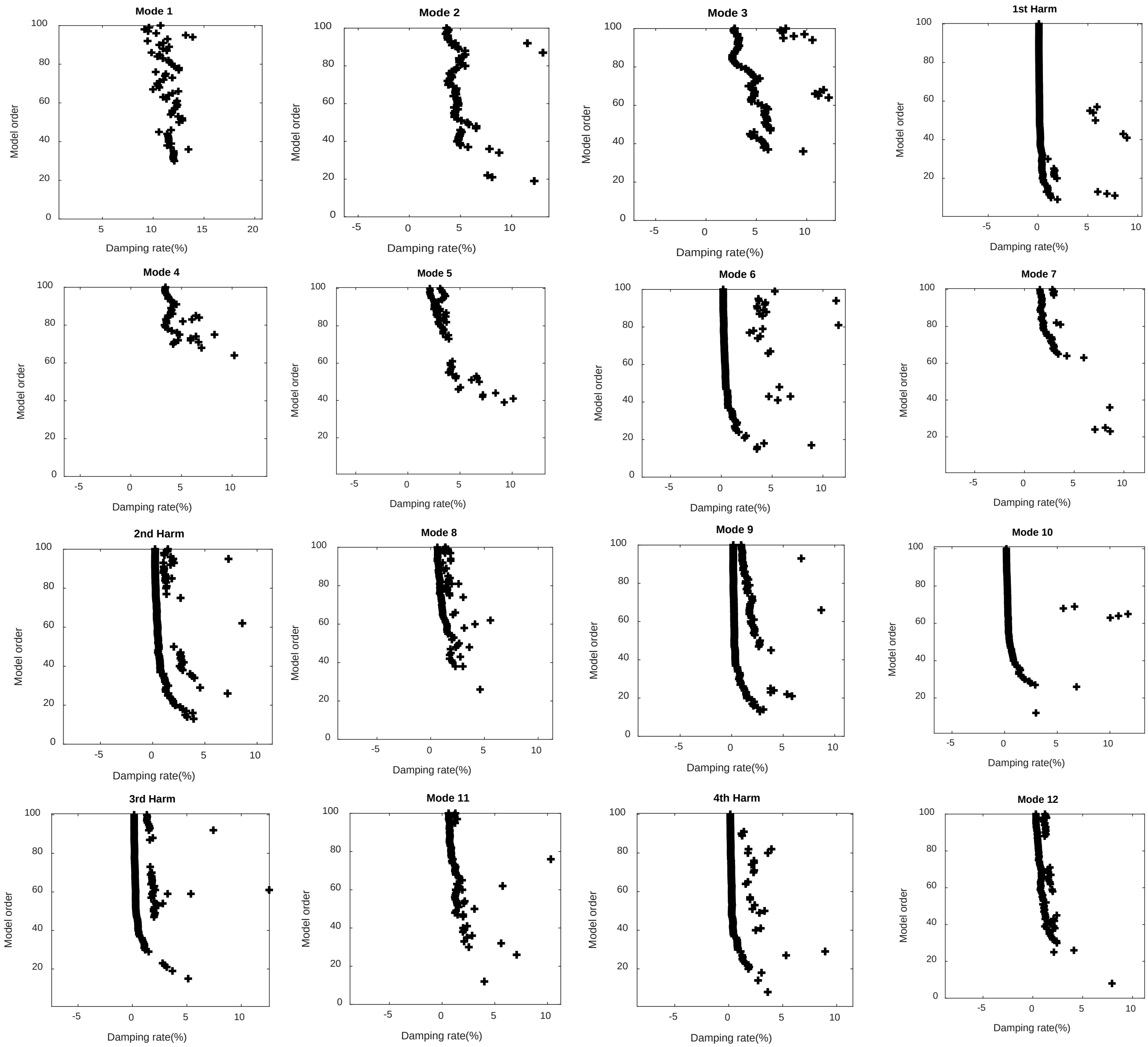

Figure 12. Damping ratio stabilization diagrams

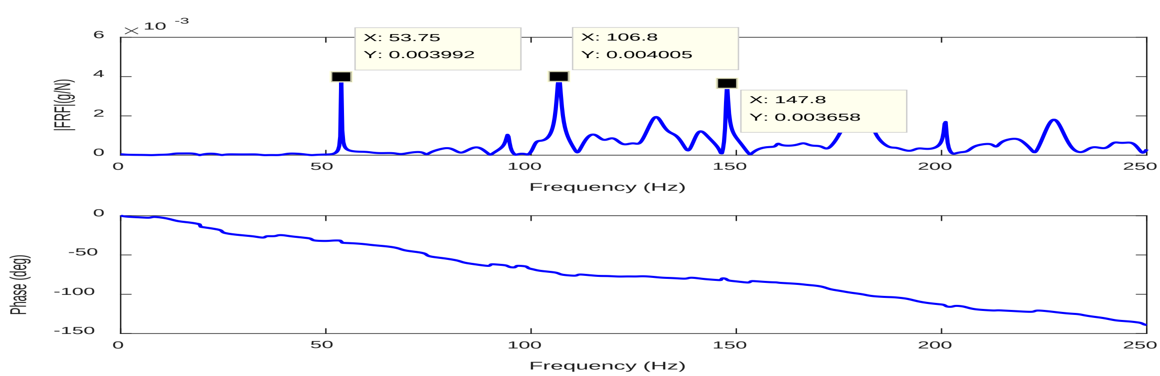

Figure 13. Identified Transfer Function FRFxx

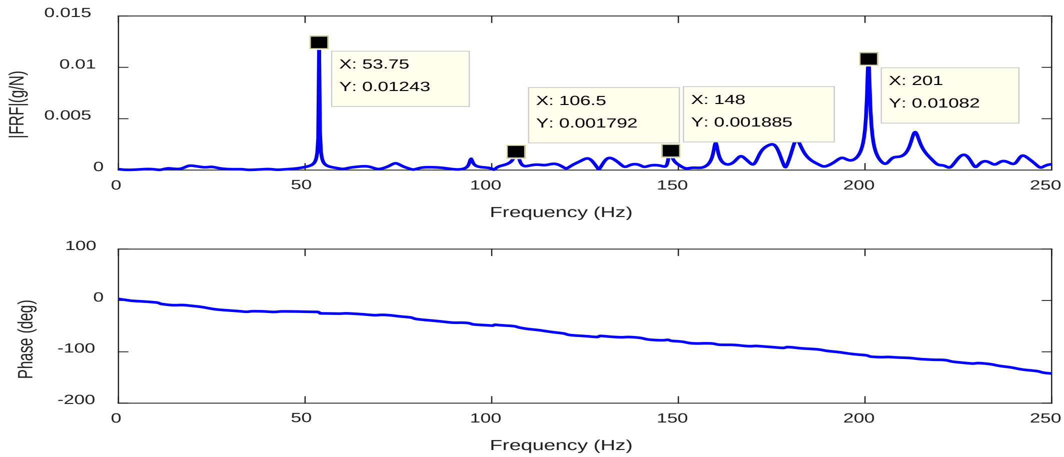

Figure 14. Identified Transfer Function FRFxy

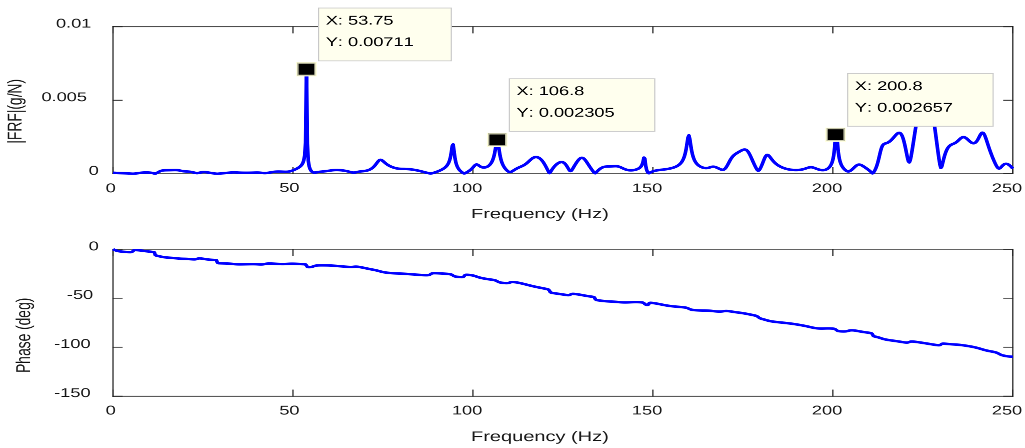

Figure 15. Identified Transfer Function FRFxz

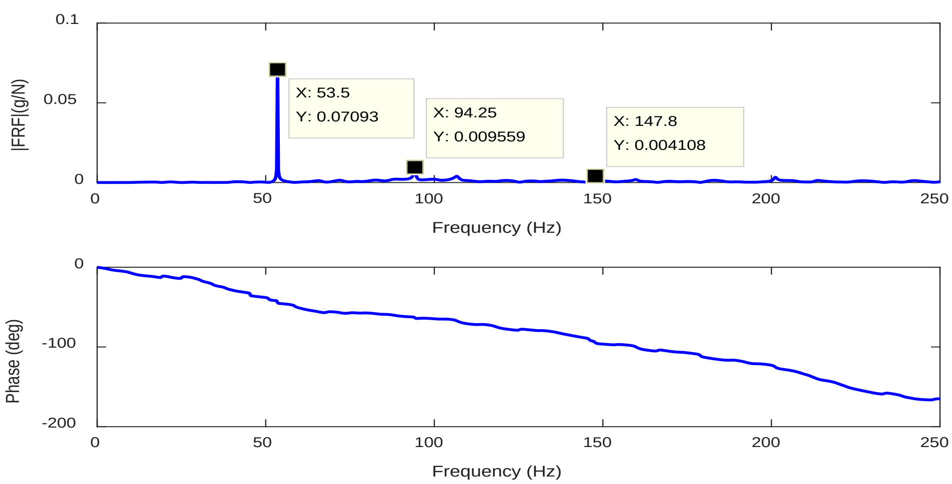

Figure 16. Identified Transfer Function FRFyx

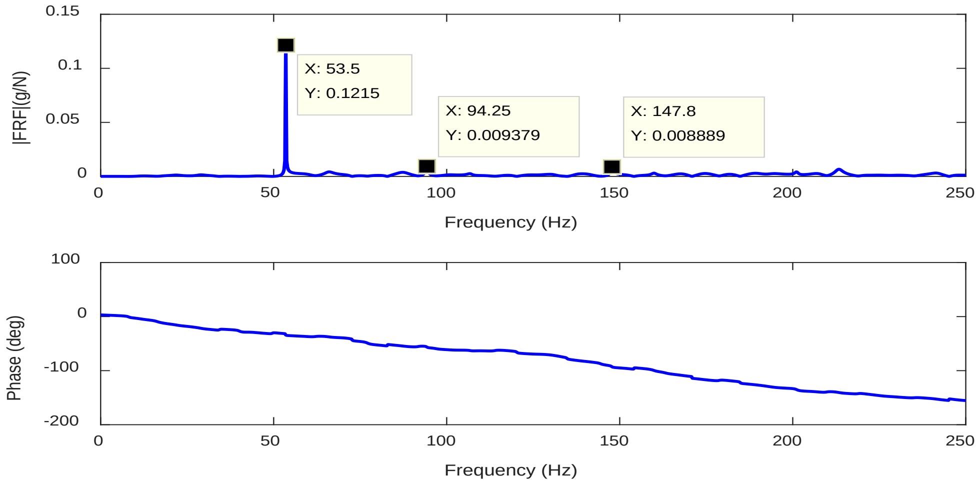

Figure 17. Identified Transfer Function FRFyy

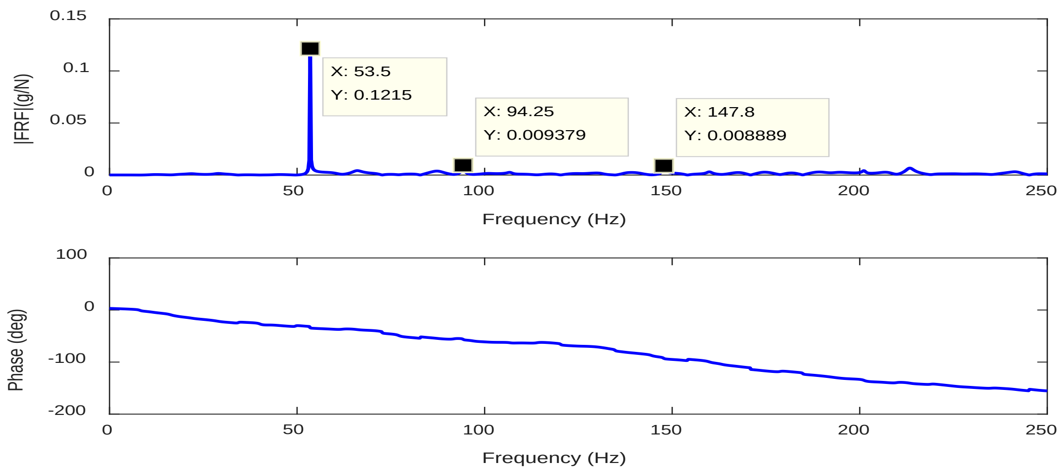

Figure 18. Identified Transfer Function FRFyz

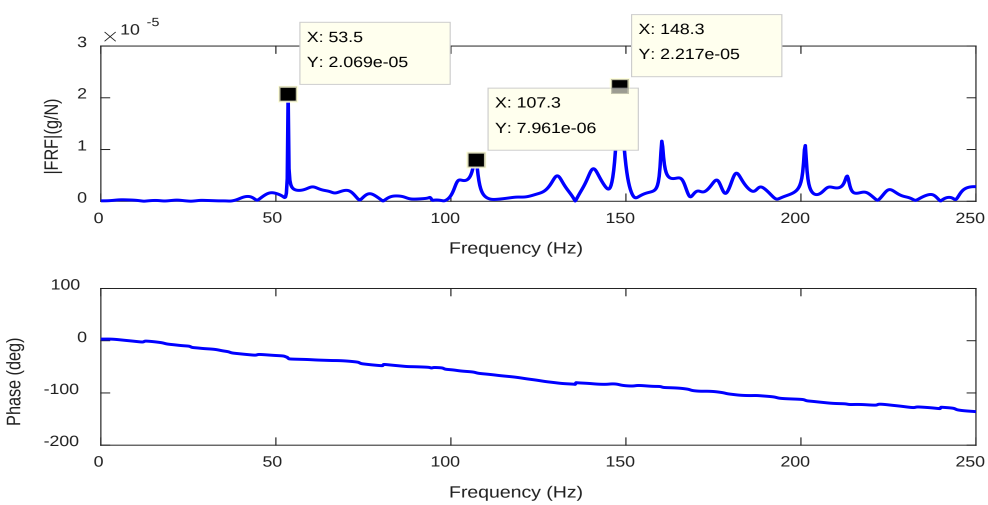

Figure 19. Identified Transfer Function FRFzx

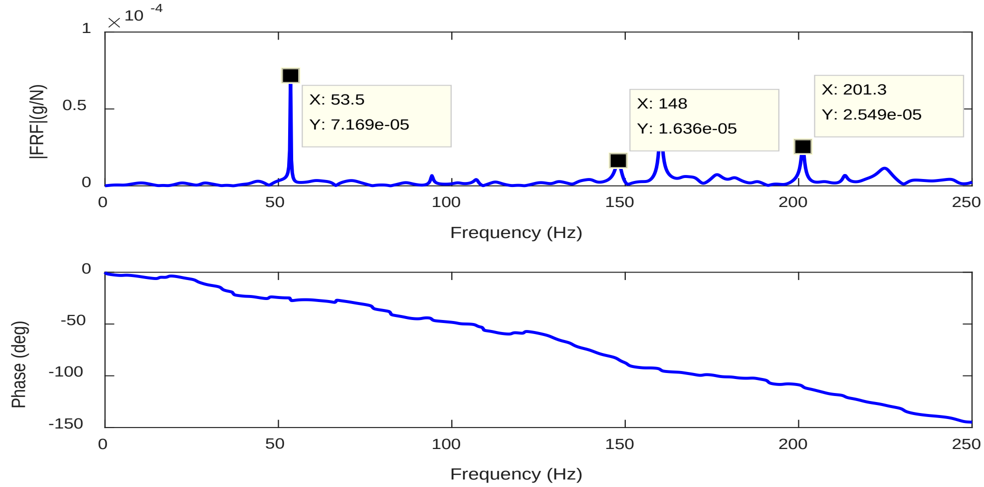

Figure 20. Identified Transfer Function FRFzy

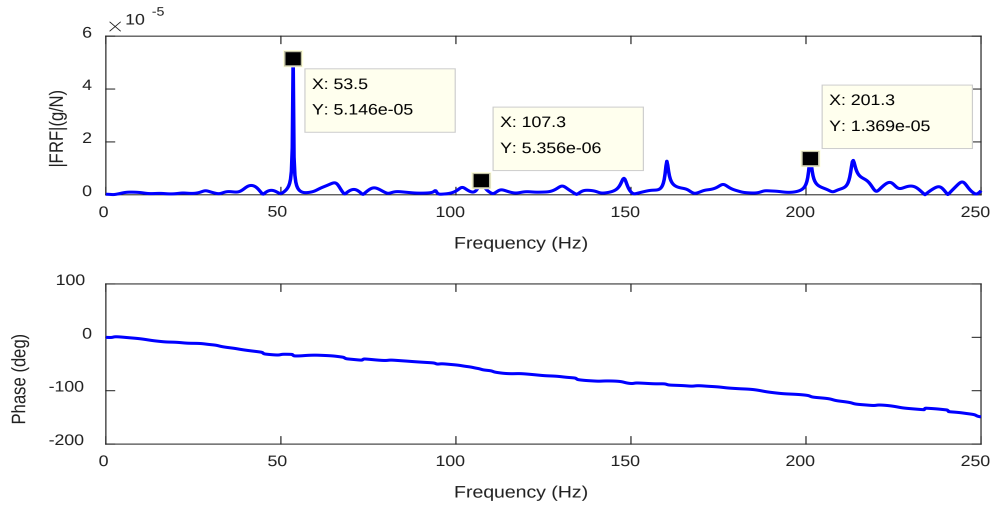

Figure 21. Identified Transfer Function FRFzz

Mode	**M 1**	**M 2**	**M 3**	**1^{st} harm**	**M 4**	**M 5**	**M 6**	**M 7**
Frequencies (Hz) - ARX	14.2	24.4	28.2	53.3	61.2	66.2	94	101.5
Frequencies (Hz) - AR	12.7	24.6	28.2	54.2	60.1	66.2	93.9	101.3
Damping (%) - ARX	11.5	3.7	3.3	0	3.4	2.7	0.2	1.5
Mode	**2^{nd} harm**	**M 8**	**M 9**	**M 10**	**3^{rd} harm**	**M 11**	**4^{th} harm**	**M 12**
Frequencies (Hz) - ARX	106.3	111.7	147.5	149.2	159.5	200.4	212.6	224.6
Frequencies (Hz) - AR	106.3	113.5	147.3	149.1	159.5	201.3	213.8	225.1
Damping (%) - ARX	0	0.6	0.2	0.2	0	0.2	0	0.7

Table 1. Identified frequencies and damping ratios

By comparison to the identified frequencies by MODALAR based on updated AR model [4] shown in table 1, the approach reveals high accuracy identified natural and harmonic frequencies with their damping ratios.

Moreover, frequency response function is directly identified from grinding operation based on ARX model. The results are better observed on the X and Y directions, this can be explained by the configuration of Scompi when working in horizontal surface to perform the grinding task.

5. Conclusion

This work is a part of an ongoing research program on investigating vibration problems of flexible manipulator. The frequencies, damping ratios and operational FRFs can be constructed and most excited modes are revealed during the grinding process. In this paper, operational FRFs of a structure are identified directly from measured signals via an ARX model. The results illustrated the sensibility of the acceleration in the X and Y directions while the contrary is proved in the Z direction with low magnitudes of the FRFs. Furthermore, as damping of the grinding process and equivalent stiffness are in command of cutting stability, so their identification is crucial to predict and avoid detrimental chatter occurrence. In the ongoing research, the inverse of ARX model will be applied in order to estimate the excitation force in the working conditions, with the integration of phase and coupling between directions. The interest lies in the reconstruction of excitation forces that gave rise to measured response signals based on ARX model. This approach is expected to serve for monitoring and vibration control design of the robot during machining operation.

Acknowledgments

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