Semi-active Torsional Vibrations Control of a Rotor Using a Smart ER Dynamic Absorber

Yulan Sun and Marc Thomas

Department of Mechanical Engineering, ETS,
1100, Notre-Dame West Street, Montreal, Quebec, (H3C 1K3), CANADA
marc.thomas@etsmtl.ca

Abstract
Torsional rotor vibration is always very difficult to control since the implementation of a control system is not an easy task while the machine is rotating. Excessive torsional vibration can lead to failures of many mechanical components. A common method for controlling vibrations involves the use of dynamic absorbers. Due to their variable properties, smart materials can be used to increase the frequency range in order to control vibration. This article is concerned with the application of Electrorheological Fluids (ERF) to the reduction of torsional vibrations of a rotor by controlling damping and stiffness of a rotational dynamic absorber. A cylindrical type of electrorheological (ER) torsional absorber is designed and manufactured according to the required damping force level and the critical velocity of the rotor system. This paper presents torsional vibration control performance of a smart ER dynamic absorber using a bang-bang (ON-OFF) control strategy. The experimental results very closely approximate the simulation results. These results show that the ER dynamic absorber exhibits very good performances in terms of reducing the torsional vibration of rotor system.

Key words: Electrorheological fluids, torsional vibrations, dynamic absorber, Bang-Bang control, smart material.

1. Introduction

Torsional rotor vibration is always very difficult to control since the implementation of a control system is not an easy task while the machine is rotating. Excessive torsional vibration can lead to failures of many mechanical components. Furthermore, in many industrial applications, torsional vibration problems may not be apparent until a failure occurs. For example, it is the case when the machine contains a gearbox which causes the torsional vibration to cross-couple into lateral vibrations [1]. A common method to control vibrations involves the use of dynamic absorbers [2]. However, when the physical parameters of dynamic absorbers are constant, their frequency range efficiency is tight and thus not really suitable for variable speeds systems. To remedy this, a damping could be added to the absorber, but this would lead to a loss of efficiency at the main frequency and increases the transmissibility on a large frequency range. On the other hand, due to their variable properties, smart materials may be used to increase the frequency range in order to control vibration. Electrorheological (ER) fluids are attractive materials that undergo very fast reversible changes in their rheological properties (mainly their yield stress and effective viscosity) upon the application of electric fields [3]. This fluid may potentially be applied in many industrial areas [4]. They may be used to control vibrations [5], to control valves [6] or rotor squeeze film dampers [7]. ER fluid may also be applied to control active suspensions [8] or seismic vibrations [9], through the application of high electric fields. Previous studies conducted by the authors have allowed the development of a new very efficient Electro-Rheological (ER) fluid [10]. In this paper, we propose the design of a new dynamic torsional absorber by using ER fluid for a rotor system. The smart absorber is controlled by varying the electric field applied to the fluid.

2. ER smart dynamic torsional absorber

2.1 Model of ER fluid

In general, most ER fluid exhibit Bingham rheological behaviors and shear-thinning. In previous studies, the authors developed a model call the Quasi-Bingham model to explain this complex phenomenon (Sun *et al*., 2009, 2010), which is:

$$\begin{aligned}
&\tau = \tau_{yQB} + \eta_\infty \dot{\gamma} \quad or \quad \tau = \eta_\infty \dot{\gamma} && when \quad E = 0 \\
&\tau = \tau_{yQB} + \eta_0 \dot{\gamma} && when \quad E \neq 0 \quad \dot{\gamma} \leq \dot{\gamma}_1 \\
&\tau = \tau_{yQB} + \eta_\infty \dot{\gamma} + (\eta_0 - \eta_\infty)\frac{\dot{\gamma}}{1+(t\dot{\gamma})^n} && when \quad E \neq 0 \quad \dot{\gamma}_1 \leq \dot{\gamma} \leq \dot{\gamma}_2 \\
&\tau = \tau_{yQB} + \eta_\infty \dot{\gamma} && when \quad E \neq 0 \quad \dot{\gamma} \geq \dot{\gamma}_2
\end{aligned} \tag{1}$$

where τ is the shear stress (Pa); $\dot{\gamma}$ is the shear rate; τ_{yQB} is the elastic yield stress (Pa) which is dependent on the strain and on the applied electric field E.

According to the dielectric loss model of Hao *et al*. [11], the yield stress τ_{yQB} is a function of E^2; η_0 is the zero shear viscosity; it is defined as the value at a very low shear rate and is a function of E; η_∞ is the infinite shear viscosity; it is defined as the value at very high shear rate; the parameter "n" is known as the Cross Rate Constant [12]. It is a dimensionless factor and is a measurement of the degree of dependence of viscosity on shear rate in the shear-thinning region; "t" is known as the Cross Time variable and has a dimension of time. The reciprocal, 1/t, gives us a critical shear rate that is a useful indicator of the onset shear rate for shear thinning; $\dot{\gamma}_1, \dot{\gamma}_2$ are two critical shear rates. We can obtain the values by using the two equations:

$$1+\left(t\dot{\gamma}_1\right)^n \approx 1; \quad \dot{\gamma}_2 \gg 1/t \tag{2}$$

This model can explain very well the rheological behaviors. An empirical equation (equation (3) of stresses was developed for a mixture called ETSERF40-20 as an example of the application of the Quasi-Bingham model. This equation is very useful for modeling a control system with the ER fluid [13].

$$\tau = \tau_{yQB} + \eta_\infty \dot{\gamma} + (\eta_0 - \eta_\infty)\frac{\dot{\gamma}}{1+(t\dot{\gamma})^n} \tag{3}$$

$$\tau = 27.04\times10^{-9} E^2 + 0.218\dot{\gamma} + 29.4\times10^{-3} E \frac{\dot{\gamma}}{1+t\dot{\gamma}} \tag{4}$$

The electric field E is expressed in kV/mm.
Equations 3 and 4 may be replaced by Equation (5) accordingly with the following relationships:

$$\tau \quad = \alpha E^2 + \eta_\infty \dot{\gamma} + \beta E \frac{\dot{\gamma}}{1 + t\dot{\gamma}} \tag{5}$$

$$\tau_{yQB} = \alpha E^2 \, ; \eta_0 = \beta E + \eta_\infty \tag{6}$$

The α , β and η_∞ are intrinsic values of the ER fluid to be experimentallv determined. The field-dependent yield stresses of these ER fluids were experimentally obtained by $\tau_{yQB} = 27.04 \times 10^{-3} E^2$ Pa. The viscosity dynamic was experimentally obtained by $\eta_0 = 29.4E + 0.218$ for fluid ETSERF40-20. This rheological model allows for the exploration of the suitability of ER fluids to control the torsional vibrations of rotors through simulations.

2.2 ER torsional absorber design

A cylindrical type of ER torsional absorber is designed and manufactured according to the required damping force level and a critical velocity of a rotor system. Figure 1 describes the prototype of the proposed ER torsional absorber, which consists in an outer cylinder for the rotor and an inner cylinder for the absorber with the ER fluid enclosed between both cylinders. The ER fluid is composed of diatomite (called ETSERF40-20) mixed into silicone oil and is used to study the effect of ER fluids on the dynamic absorber [10] , [13].

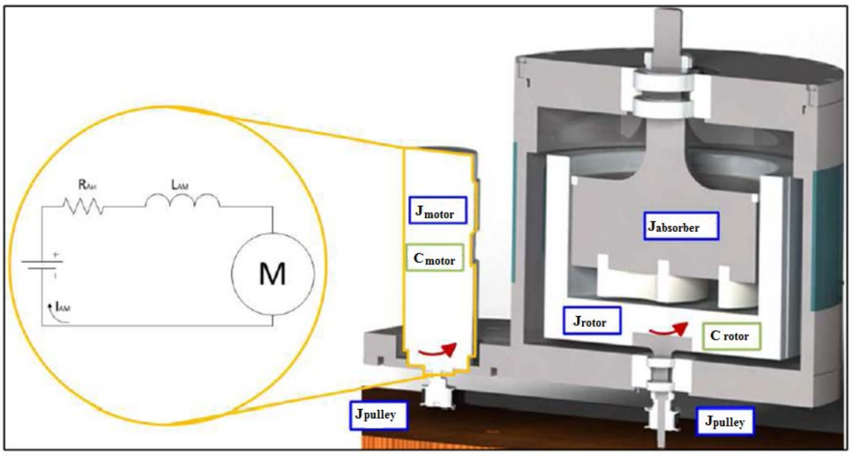

Figure 1. Overview of the torsional dynamic absorber

The positive (+) voltage is connected to inner cylinder and the negative voltage (-) connected to the outer cylinder. The proposed ER absorber is detailed accordingly with the following design parameters; electrode length (H)= 62.5mm and electrode gap (e)= 4.4mm. In the absence of electric field, the ER absorber produces a damping force only due to the fluid resistance when rotating. This damping (called C_2) of ER fluid can be calculated from equation (7) [13]. It is inherent, and is not influenced by the electric field.

$$C_2 = \frac{2\pi R^3 H \eta_\infty}{e} \tag{7}$$

R is the internal radius of outer cylinder, η_∞ is the viscosity at infinite velocity that is intrinsic value of the ER fluid to be experimentally determined. Applied to the considered fluid ETSERF40-20, we found C_2 = 0.0092 N.m.s/rad.

If a certain level of the electric field is applied to the ER absorber, the ER absorber produces an additional damping force. This damping force can be continuously tuned by controlling the intensity of the electric field. This system is then called semi-active.

2.3 Mechanical model of the smart system

2.3.1 Dynamic mode of a rotor system with the ER absorber

The vibration of the initial system may be described as a single degree-of-freedom (SDOF) damped rotating system, composed of the rotational inertia J_1 = 0.116 kg.m^2, the torsional stiffness K_1 = 801.51 N.m/rad, and the torsional damping C_1 =2.192 N.m.s/rad. The theoretical natural frequency of this primary system is 13.23 Hz with a damping rate of 11.37%. It is the amplitude close to this frequency that must be controlled.

This vibration control method in this system consists in adding another SDOF system, which is the torsional dynamic absorber. The smart ER dynamic absorber is composed of a rotational inertia J_2 = 0.0265kgm^2 driven by the ER fluid, which can thus be modeled by a torsional spring K_2 , a linear viscous dashpot C_2 and a viscous dashpot C_{ER} . The resulting mechanical system is a two DOF system which is illustrated in Figure 2.

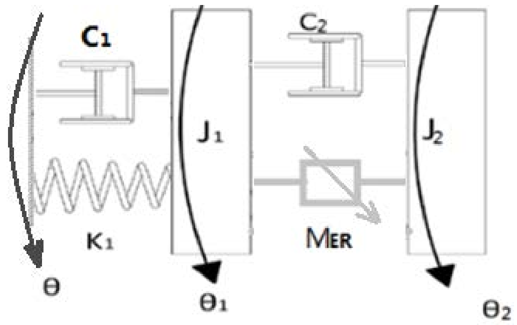

Figure 2. ER torsional dynamic absorber

The torque M_{ER} which is due to the effect of the ER fluids is composed of a nonlinear torsional stiffness torque and a nonlinear viscous torque, which depend on the electric field. It is only active when the electric field is applied. It can be expressed as:

$$M_{ER} = C_{ER}(\dot{\theta}_1 - \dot{\theta}_2) + K_{ER}(\theta_1 - \theta_2) \tag{8}$$

The total torque of the proposed ER absorber can be obtained as $M_F = C_2\dot{\theta} + M_{ER}$, where C_2 is the effective damping due to the fluid viscosity, θ is the excitation angular displacement, and M_{ER} is the field-dependent damping torque which is tunable as a function of the applied electric field.

The rotor dynamic system could be described as follows:

$$\begin{aligned} J_1\ddot{\theta}_1 &= -M_{ER} - C_1(\dot{\theta}_1 - \dot{\theta}) - K_1(\theta_1 - \theta) - C_2(\dot{\theta}_1 - \dot{\theta}_2) \\ J_2\ddot{\theta}_2 &= -C_2(\dot{\theta}_2 - \dot{\theta}_1) + M_{ER} \end{aligned} \tag{9}$$

2.3.2 Modeling of the system by state space formulation

Equation 9 can be expressed as the following equations:

$$\begin{aligned}
&J_1\ddot{\theta}_1 + K_1\theta_1 + C_1\dot{\theta}_1 + C_2(\dot{\theta}_1 - \dot{\theta}_2) = K_1\theta + C_1\dot{\theta} - M_{ER} \\
&J_2\ddot{\theta}_2 + C_2(\dot{\theta}_2 - \dot{\theta}_1) = M_{ER} \\
&\ddot{\theta}_1 = \frac{1}{J_1}(K_1\theta + C_1\dot{\theta} - M_{ER} - K_1\theta_1 - C_1\dot{\theta}_1 - C_2(\dot{\theta}_1 - \dot{\theta}_2)) \\
&\ddot{\theta}_2 = \frac{1}{J_2}(M_{ER} - C_2(\dot{\theta}_2 - \dot{\theta}_1))
\end{aligned} \tag{10}$$

By modifying the variables as follows:

$$\begin{aligned}
&x_1 = \theta_1 - h_0\theta\,; \\
&x_2 = \dot{x}_1 - h_1\theta = \dot{\theta}_1 - h_0\dot{\theta} - h_1\theta \\
&x_3 = \theta_2\,; \\
&x_4 = \dot{x}_3 = \dot{\theta}_2
\end{aligned} \tag{11}$$

We can write:

$$\theta_1 = x_1 + h_0\theta\,;\ \dot{x}_1 = x_2 + h_1\theta \tag{12}$$

We obtain:

$$\dot{x}_1 = x_2 + h_1\theta \tag{13}$$

$$\begin{aligned}
\dot{x}_2 &= \ddot{\theta}_1 - h_0\ddot{\theta} - h_1\dot{\theta} = \frac{1}{J_1}(K_1\theta + C_1\dot{\theta} - M_{ER} - K_1\theta_1 - C_1\dot{\theta}_1 - C_2(\dot{\theta}_1 - \dot{\theta}_2)) - h_0\ddot{\theta} - h_1\dot{\theta} \\
&= \frac{1}{J_1}(K_1\theta + C_1\dot{\theta} - M_{ER} - K_1\theta_1 - (C_1 + C_2)\dot{\theta}_1 + C_2\dot{\theta}_2) - h_0\ddot{\theta} - h_1\dot{\theta} \\
&= \frac{1}{J_1}\left\{K_1\theta + C_1\dot{\theta} - M_{ER} - K_1(x_1 + h_0\theta) - (C_1 + C_2)(x_2 + h_0\dot{\theta} + h_1\theta) + C_2x_4\right\} - h_0\ddot{\theta} - h_1\dot{\theta} \\
&= \frac{1}{J_1}\{K_1 - K_1h_0 - (C_1 + C_2)h_1\}\theta + \left\{\frac{1}{J_1}\left[C_1 - (C_1 + C_2)h_0\right] - h_1\right\}\dot{\theta} - h_0\ddot{\theta} \\
&+ \frac{1}{J_1}\{-M_{ER} - K_1x_1 - (C_1 + C_2)x_2 + C_2x_4\}
\end{aligned}$$

Take $h_0 = 0$ and $\frac{1}{J_1}\left[C_1 - (C_1 + C_2)h_0\right] - h_1 = 0$, we can obtain $h_1 = \frac{C_1}{J_1}$

We have:

$$\begin{aligned}
\dot{x}_1 &= x_2 + h_1\theta = x_2 + \frac{C_1}{J_1}\theta \\
\dot{x}_2 &= \frac{1}{J_1}\{K_1 - K_1h_0 - (C_1 + C_2)h_1\}\theta + \left\{\frac{1}{J_1}\left[C_1 - (C_1 + C_2)h_0\right] - h_1\right\}\dot{\theta} - h_0\ddot{\theta} \\
&+ \frac{1}{J_1}\{-M_{ER} - K_1x_1 - (C_1 + C_2)x_2 + C_2x_4\} \\
&= \frac{1}{J_1}\left\{K_1 - (C_1 + C_2)\frac{C_1}{J_1}\right\}\theta - \frac{1}{J_1}M_{ER} + \frac{1}{J_1}\{-K_1x_1 - (C_1 + C_2)x_2 + C_2x_4\} \\
\dot{x}_3 &= x_4
\end{aligned} \tag{14}$$

$$\dot{x}_4 = \frac{C_1 C_2}{J_1 J_2}\theta + \frac{1}{J_2}M_{ER} + \frac{1}{J_2}\left\{C_2 x_2 - C_2 x_4\right\}$$

By regrouping these equations under a matrix form, we obtain:

$$\begin{pmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0\\ -\frac{K_1}{J_1} & -\frac{(C_1+C_2)}{J_1} & 0 & \frac{C_2}{J_1}\\ 0 & 0 & 0 & 1\\ 0 & \frac{C_2}{J_2} & 0 & -\frac{C_2}{J_2}\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix} + \begin{pmatrix} \frac{C_1}{J_1} & 0\\ \frac{1}{J_1}\left\{K_1 - (C_1+C_2)\frac{C_1}{J_1}\right\} & -\frac{1}{J_1}\\ 0 & 0\\ \frac{C_1C_2}{J_1J_2} & \frac{1}{J_2}\end{pmatrix}\begin{pmatrix}\theta\\ M_{ER}\end{pmatrix} \tag{15}$$

$$x = \begin{pmatrix}x_1 & x_2 & x_3 & x_4\end{pmatrix}^T$$

$$\dot{x} = \begin{pmatrix}\dot{x}_1 & \dot{x}_2 & \dot{x}_3 & \dot{x}_4\end{pmatrix}^T$$

$$u = \begin{pmatrix}\theta & M_{ER}\end{pmatrix}^T$$

By calling these matrices as:

$$y = \begin{pmatrix}0 & 1 & 0 & 0\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\\ x_4\end{pmatrix} \tag{16}$$

$$A = \begin{pmatrix} 0 & 1 & 0 & 0\\ -\frac{K_1}{J_1} & -\frac{(C_1+C_2)}{J_1} & 0 & \frac{C_2}{J_1}\\ 0 & 0 & 0 & 1\\ 0 & \frac{C_2}{J_2} & 0 & -\frac{C_2}{J_2}\end{pmatrix}, \quad B = \begin{pmatrix} \frac{C_1}{J_1} & 0\\ \frac{1}{J_1}\left\{K_1 - (C_1+C_2)\frac{C_1}{J_1}\right\} & -\frac{1}{J_1}\\ 0 & 0\\ \frac{C_1C_2}{J_1J_2} & \frac{1}{J_2}\end{pmatrix}$$

$$C = \begin{pmatrix}0 & 1 & 0 & 0\end{pmatrix}$$

We obtain the classic state space formulation:

$$\dot{x} = Ax + Bu$$
$$y = Cx$$

We consider a simple and linear case, in order to establish an ON-OFF control. The block diagram is shown in Fig. 3.

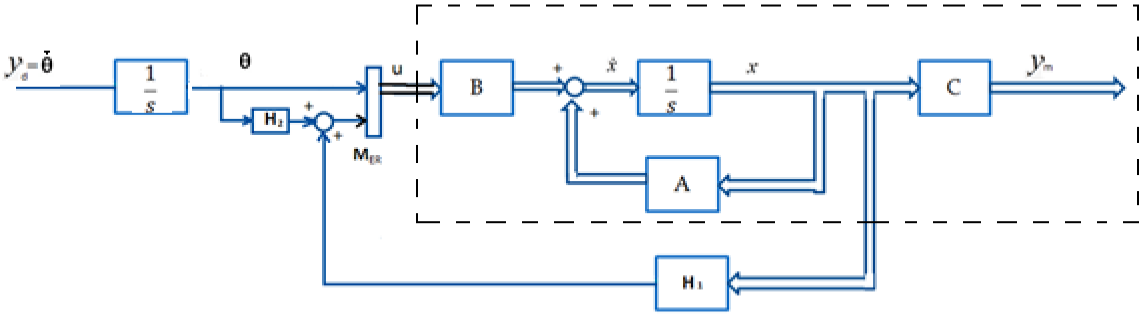

Figure 3. Block diagram du system

By defining

$$H_1 = \left(K_{ER} \quad C_{ER} \quad -K_{ER} \quad -C_{ER} \right); \quad H_2 = \frac{C_{ER}C_1}{J_1}; \tag{17}$$

The controllable torque may be expressed as:

$$\begin{aligned} M_{ER} &= K_{ER}(x_1 - x_3) + C_{ER}(x_2 - x_4 + \frac{C_1}{J_1}\theta) \\ M_{ER} &= K_{ER}x_1 - K_{ER}x_3 + C_{ER}x_2 - C_{ER}x_4 + \frac{C_{ER}C_1}{J_1}\theta = H_1 x + H_2\theta \end{aligned} \tag{18}$$

H_1 *and* H_2 can be controlled by the electrical field by checking the difference of the angular velocity between $y_d = \dot{\theta}$ and $y_m = \dot{\theta}_m$. If the difference of the absolute value of is greater than zero, we must apply the electric field; otherwise we set the electric field to zero.

3. Numerical simulation

The control system with an Dynamic absorber (SERDA) has been numerically simulated by using Simulink (figure 4), in order to observe the reaction of the system, we use a linear chirp signal which is sine wave whose frequency varies linearly with time (figure 5), the amplitude is 1. We select E = 0 and 1 kv/mm in this simulation, the results is showed in the figure 6. We can see that amplitude of the response varies with the frequency; the amplitude is very small when the frequency is down 13Hz or more than 13Hz, but it arrives the maximum value, when the frequency is around 13Hz without electric fields and with the electric field, the amplitude decreases to about half of the original amplitude when electric fields applied is 1 kv/mm.

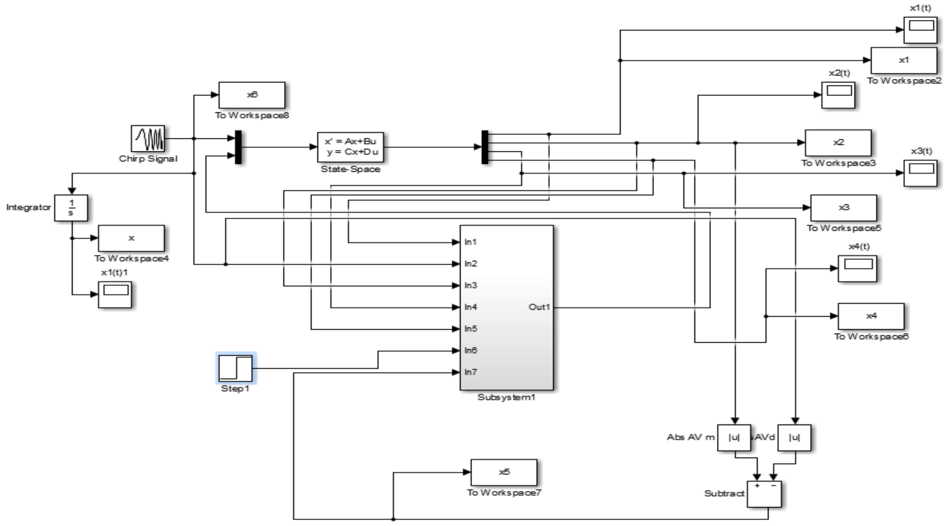

Figure 4. Block Simulink

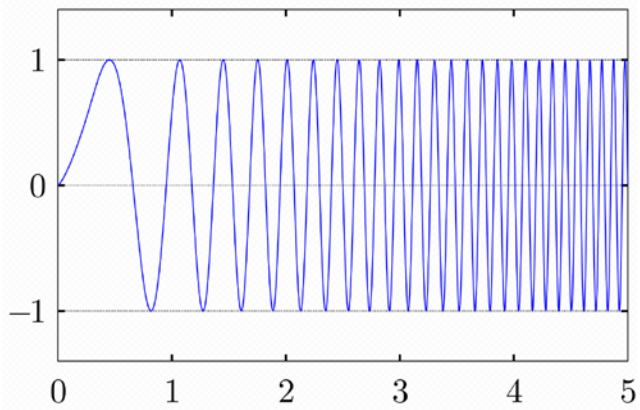

Figure 5. Linear chirp waveform

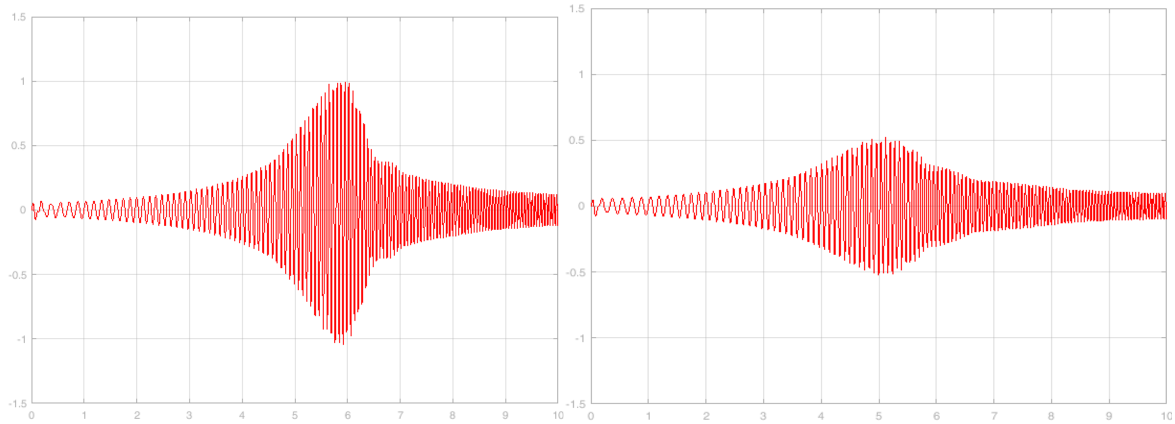

Figure 6.Reponse of system under a chirp signal obtained by simulation (left figure: E=0kV/mm; right figure : E-1kV/mm)

4. Experimental verification

In this system, the two cylinders J_1 and J_2 serve as electrodes for the ER effect in this study. The oscillation caused by the torsional vibration is measured by a triaxial accelerometer placed in the radial position as shown in Figure.6.

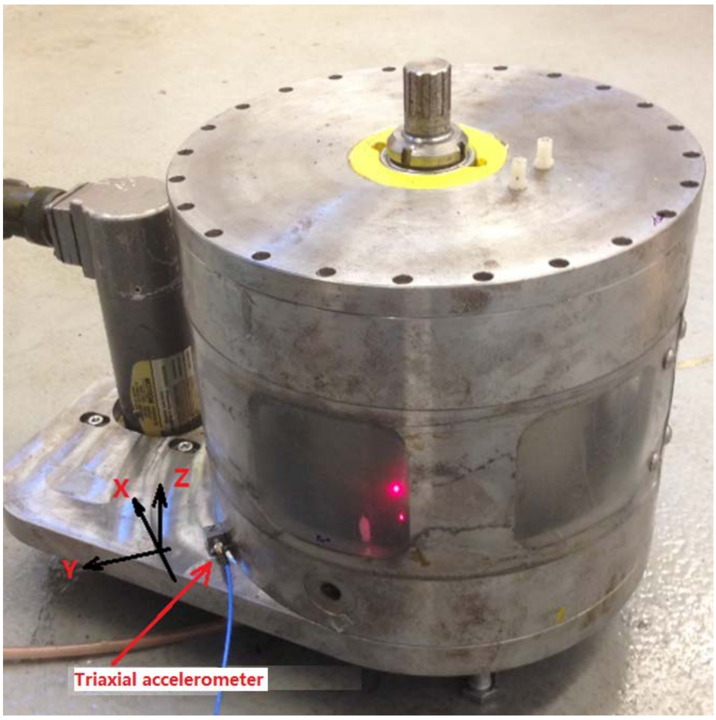

Figure 7. Electrorheological torsional absorber

We clearly see that vibration control performance in figures 8, 9 and 10, especially at 13 Hz, since the natural torsional frequency of the system is designed for this value as 13.24Hz, the critical angular velocity is 794 RPM. The two peaks as shown in Figure 8 are two natural frequencies, one is of flexible vibration, and the other is of vibration torsional. In fact, this absorber is designed only to attenuate the torsional vibration to determine the effects of the fluid ER in torsional vibration. Clearly, the torsional vibrations are attenuated in the range of 12.5 to 15Hz, but the peak which is at 12Hz does not change anything, it shows that this absorber is effective for torsional vibration, but not for flexible vibration. It can be noticed that the results of simulation are very comparable with the experimental results.

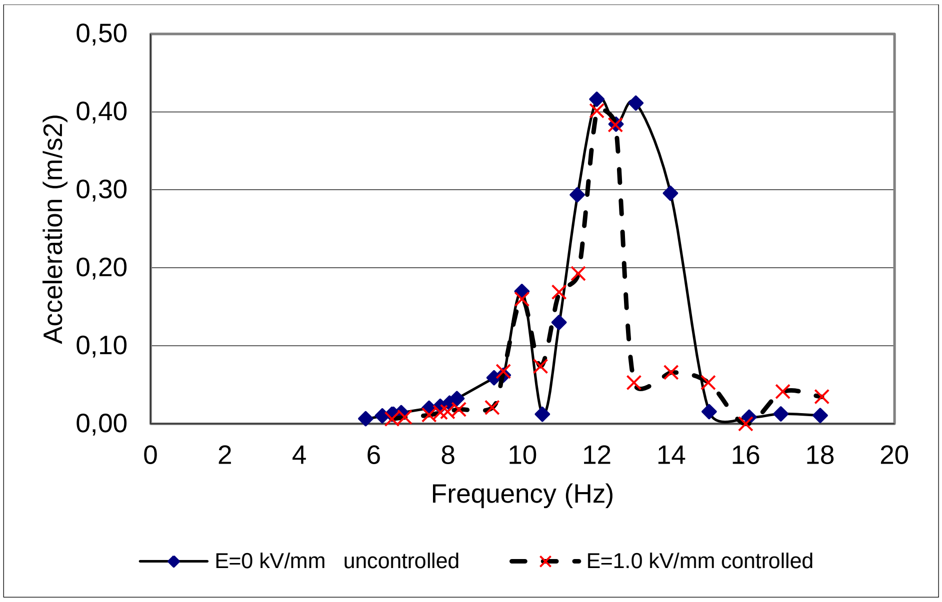

Figure 8.Response acceleration of system under different electrical field

The measurement results in time and frequency domain are shown in figure 9 and 10 for different applied electric fields. The amplitude of sinusoidal vibration decreases when the applied field increases, but not for the high electric field as E = 1.36 kv / mm since there is an optimal value of electric fields that applies on ER fluid to attenuate the torsional vibrations when a simple On-Off control strategy is chosen to realize this control system.

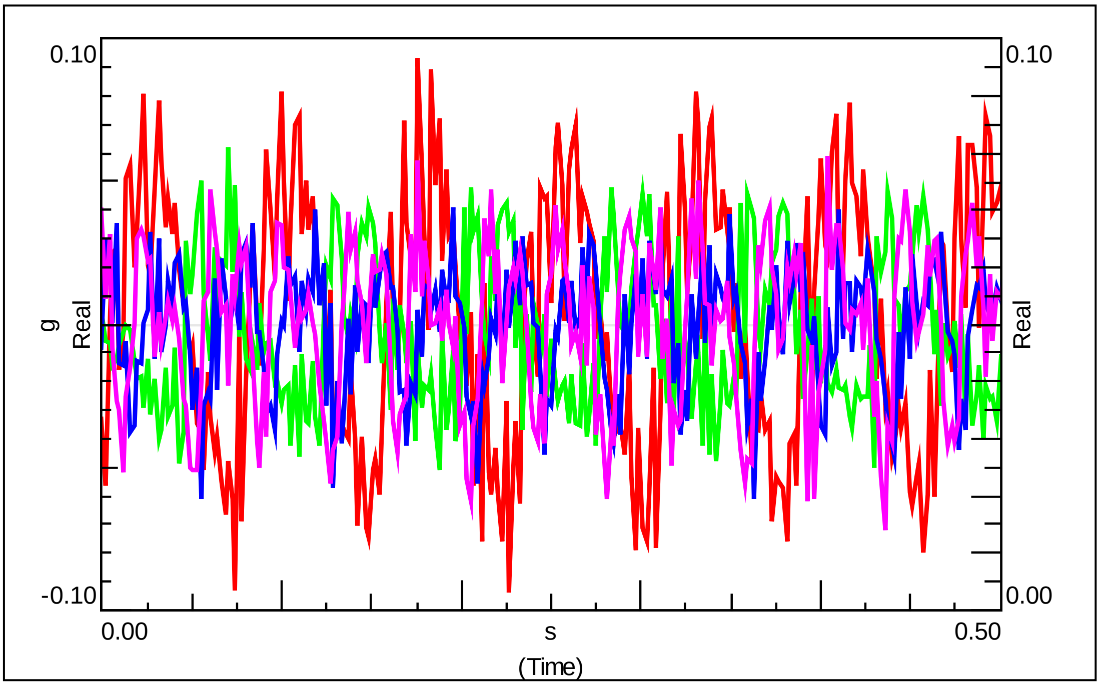

Figure 9.Temporal response of system when the rotor rotates in 780 RPM under different electrical field (red: E=0kV/mm; green: E=0.68 kV/mm; blue: E=1kV/mm; pink: E=1.36kV/mm)

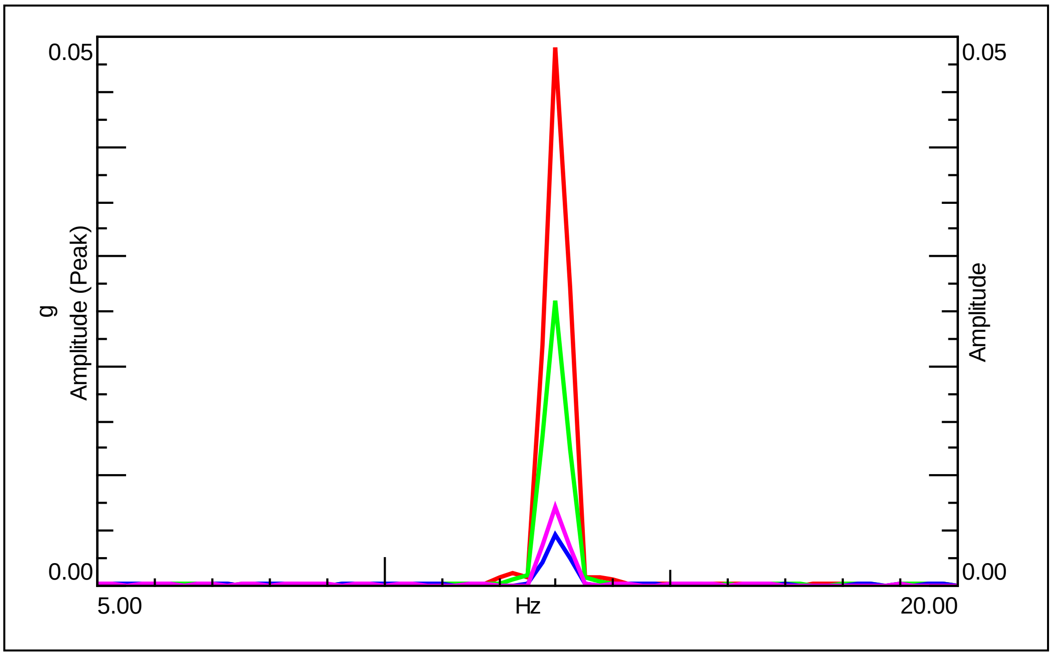

Figure 10. Frequency response of system when the rotor rotates in 780 RPM under different electrical field (red: E=0kV/mm; green: E=0.68 kV/mm; blue: E=1kV/mm; pink: E=1.36kV/mm)

5. Conclusion

This article is concerned with the application of Electrorheological Fluids (ERF) to the reduction of torsional vibrations of a rotor by controlling damping and stiffness of a rotational dynamic absorber. A cylindrical type of electrorheological (ER) torsional absorber was designed and manufactured according to the required damping force level and the critical velocity of the rotor system. This paper presents torsional vibration control performance of a smart ER dynamic absorber using a bang-bang (ON-OFF) control strategy. The absorber efficiency is measured, and the results show that the ER dynamic absorber exhibits very good performances in terms of reducing the torsional vibration of rotor system. The experimental results very closely approximate the simulation results. By use of controlled damping one can reduce (minimize) unwanted torsional vibration not only in theory but also in practice, this study shown that ER fluid are useful smart materials to realize a controllable system.

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