Measurement and use of transmission error for diagnostics of gears

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Abstract

Transmission error (TE) has long been thought to be a major contributor to gear vibration and noise, but insufficient consideration has been given to the different types of TE and how they generate vibrations. TE is defined as the difference in torsional vibration of two meshing gears, scaled so as to represent linear motion along the line of action. There are three distinct types of TE; 1) Geometric TE (GTE) given by deviations of the (combined) tooth profiles from ideal involute; 2) Static TE (STE) including elastic deformation of the teeth and therefore being load dependent; 3) Dynamic TE (DTE) including inertial as well as stiffness effects, and thus being speed as well as load dependent. It has long been recognized that TE can be measured very accurately by phase demodulation of the signals of shaft encoders rigidly attached to each of the gears in mesh, but only recently realized that all three types can be measured; GTE at low speed and low load, STE at low speed and higher load, and DTE at higher speed and higher load. This paper demonstrates that TE has several advantages over vibration acceleration (or even the raw torsional vibrations) as a diagnostic parameter, being close to the source (the gearmesh) and with “common mode rejection” from the two gears, thus being much less sensitive to operating conditions and rig parameters, including the much greater number of transfer paths, modulations, and resonances in the casing vibration measurements. The measurements in this paper were made on a single stage gearbox, over an input gear speed range from 2 – 20 Hz, and input shaft torque range from 0 – 20 Nm. Earlier measurements on the same gearbox were for soft gears which developed distributed pitting over an operating period of many hours. Unfortunately, the encoders used at that time (actually included in slip rings) had a low torsional resonance frequency, which precluded obtaining TE at higher than 2 Hz shaft speed, so only GTE and STE could be estimated. New results are presented here for ground, hardened gears with a simulated tooth root crack on one tooth. Not only does this illustrate the differences with a local fault, but new encoders were mounted, valid up to a shaft speed of 20 Hz, so that DTE could also be measured.

1 Introduction

Gear transmission error (TE) is defined as the difference in torsional vibration of two gears in mesh, scaled so as to represent linear motion along the line of action, this being common to the two gears. Already in 1996 [1], it was shown that TE could be measured simply and accurately by phase demodulation of the pulse signals from high quality shaft encoders on the free ends of the shafts on which the gears are mounted. The measured torsional vibrations, in terms of angular displacement, are scaled by the respective base circle radii, and subtracted to give relative motion along the line of action. The accuracy of the encoders themselves corresponds to fractions of a micron of TE, and virtually no further error is introduced by the phase demodulation processing by Hilbert transform techniques (as compared with the earlier use of analogue phase meters, or polynomial interpolation between pulses). It is often possible to mount the encoders on the free ends of the gear shafts (the section not transmitting torque) so that they follow the gear motions up to a very high frequency. The proposed application in [1] was to the measurement of TE in design, development and manufacture, to add to information gained from measurements using gear metrology machines, but it has also been proposed as a tool for gear diagnostics in [2]. However, at the time, that was limited by the necessity to mount encoders on the machines.

It is now becoming more common for encoders to be built into machines, to provide valuable information for both control and monitoring of, for example, variable speed machines such as wind turbines, and this will presumably increase with the adoption of the Internet of Things, so it is likely that measurement of TE will become more available as an indicator of gear faults.

Transmission error (TE) has long been thought to be a major contributor to gear vibration and noise, but the relationship between them has not been fully understood. For a start there are three distinct types of TE: 1) Geometric TE (GTE) given by deviations of the (combined) tooth profiles from ideal involute; 2) Static TE (STE) including elastic deformation of the teeth, and therefore being load dependent; 3) Dynamic TE (DTE) including inertial as well as stiffness effects, and thus being speed as well as load dependent.

Measurement and application of these three types of TE as a diagnostic tool were discussed in [3], but it was found that the encoders used there (actually included in slip rings) had a low resonance frequency, which precluded measurements at high enough speed to give DTE. The same test rig has now been equipped with high quality encoders, and the current paper uses new measurements with that system. Another difference is that the old measurements were made with soft gears, run for an extended period so that (uniformly distributed) pitting developed, but no distinct local faults. The current paper uses measurements made with hardened ground gears, but with a simulated tooth root crack seeded in one tooth on the pinion, to give information on local faults, and tooth root cracks in particular, this being one of the most critical faults, and most important to distinguish from less critical faults such as local spalls.

2 Test rig and measurements

The overall layout of the spur gear test rig is shown in Figure 1.

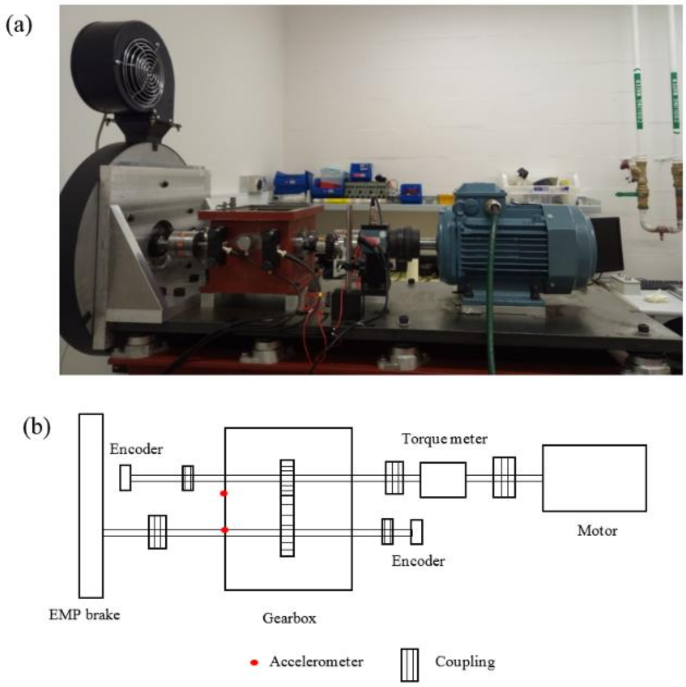

Figure 1: The spur gear test rig at UNSW. (a) photo; (b) schematic diagram

For the original measurements in [3] the reduction ratio was 19:52, and the gears were of mild steel. The original encoders were also slip rings, and had a low frequency resonance so that the highest valid input speed was 2 Hz.

For the new measurements, the reduction ratio was changed to 27:44 (same centre distance) and the gears were of hardened steel to avoid surface distress. The encoders were replaced by Heidenhain type ROD426, with 1000 pulses per rev, as well as a one per rev tacho pulse as a phase marker, and they gave valid results up to at least 20 Hz shaft speed. An EDM-generated half-tooth root crack (a 45° slot across the entire facewidth, 2.86 mm deep, extending to the tooth centreline, and 0.35mm wide) was seeded on one pinion tooth (input gear). Measurements were made at speeds 2, 5, 10, 15, 20 Hz, and loads 0, 5, 10, 20 Nm (all referenced to the input pinion). In addition to the encoder and tacho recordings, accelerometer measurements were made in the vertical direction on the casing above the input shaft at the motor end, and above both shafts at the brake end.

3 Results and discussion

3.1 Earlier results from the spur gearbox

A short summary is given here of the results published in [3], because they contain some findings which are different from those of the more recent measurements, published for the first time here. As mentioned, the gears were of mild steel and were run for a long period (nearly 50 hours) during which time they developed surface pitting fairly uniformly distributed around the gears. This was much more pronounced on the 19 tooth pinion than on the 52 tooth gear, because each tooth had a much greater number of contacts in inverse ratio to the tooth numbers, so only the pinion is discussed here.

Wear was monitored by trending the amplitude of the TE gearmesh harmonics (and the corresponding component of the synchronously averaged TE signal) in two conditions: low speed-low load (GTE) and low speed-high load (STE).

The effect of wear on GTE and STE showed an unexpected trend. The growth of the gearmesh harmonics was more pronounced on GTE during the first 6 hours of operation (mild pitting), and on STE later (severe pitting). The greater sensitivity of GTE in the initial phase was interpreted as being due to the fact that the unloaded GTE would have been dominated by (a few) local high spots at the edges of the pits, which would be easily deformed under relatively light load to give a reduced STE. On the other hand, with severe pitting more continuously distributed along the contact line, high spots would reduce the visibility of wear in GTE, and increased load would tend to give an increase in TE. Figure 2 shows a schematic representation of this interpretation, together with snapshots of the surfaces after about 2.5 and 42.5 hours of operation. For a detailed description of this test campaign the reader is referred to [4].

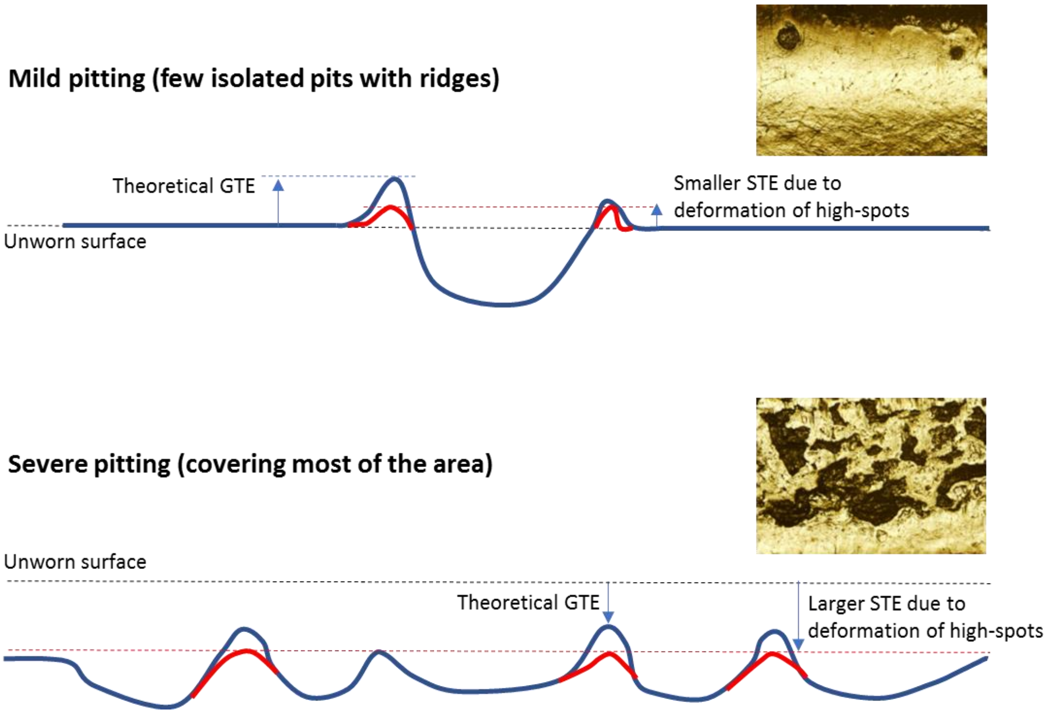

Figure 2: Schematic example of the interpretation of the effect of mild (top) and severe (bottom) pitting on GTE and STE, with corresponding example images of the gear surface.

In simulation models it is quite common to have GTE as a fixed value in series with the toothmesh stiffness. The latter is not always constant, but any nonlinearity is usually taken to correspond just to the extra compliance of the Hertzian component at low load, which still does not give a large difference in the overall stiffness, since the Hertzian component typically only represents about 25% of the total compliance, with the dominant bending stiffness component being almost linear. The above experience with “high points” does seem to indicate that, to obtain a reasonable match between such a simplified model and experiment, it would be better to use a value of GTE measured at a low, but non-zero, load sufficient to negate the effect of the high spots, and giving a more sudden transition to the Hertzian affected section of the stiffness curve.

Another interesting finding from the same study showed that, differently from TE, vibration was almost entirely insensitive to wear in both unloaded and loaded cases, at low speed. This was attributed to the fact that the proportion of the STE due to tooth deflection is still relatively small, but in fact it is only the dynamic tooth load, giving this deflection, which gives rise to vibration. At low speed there is no inertial resistance to rotation, so the driven gear can simply absorb the GTE by relative torsional motion, with almost no change in the GM spring force, even for the loaded case where the static load is almost constant. It could be expected that for DTE the much greater angular accelerations involved might prevent the driven gear from simply "moving out of the way" and thus force tooth deflection and increased vibration. This was actually found in [3] for the higher harmonics of gearmesh. Unfortunately, the encoders mounted at the time of this first test had a low resonance preventing reliable measurements of TE at speeds higher than 2 Hz (i.e. DTE) and their comparison with the vibration.

3.2 New results from the spur gearbox

As mentioned above, the new measurements were for a different gear ratio, and the gears were hardened and ground, to mitigate against surface distress. Moreover, they were reduced in face-width from 20 mm to 5 mm to reduce the gearmesh stiffness proportionately. The tests are to check the effects of the simulated half tooth-root crack described in section 2. It should be noted that the gearbox test rig is non-ideal (and non-typical) because the shafts are relatively long and slender (to give access inside the casing), but this means that the TE tends to be dominated by shaft deflections rather than tooth defections, making it difficult to detect changes in tooth stiffness, such as result from a crack. The tooth stiffness is at least an order of magnitude greater than the shaft stiffness. Both TE and vibration acceleration were measured over a range of speeds and loads, but speeds of 2 Hz and 20 Hz, and loads from zero (nominal) to 20 Nm are presented here. There was a small friction load corresponding to nominal zero, which was sufficient to keep the gears in contact, and allow measurement of the GTE at low speed.

Figure 3 shows the measured TE, synchronously averaged with respect to the pinion, for loads of 0, 5, 10, 15 and 20 Nm, for four different conditions:

1) Original TSA at 2 Hz
2) Original TSA at 20 Hz
3) Filtered TSA at 2 Hz
4) Filtered TSA at 20 Hz

Two (identical) rotational periods are shown. Bandpass filtering was performed to remove the masking effect of the gearmesh (GM) components and the first two harmonics of the input shaft speed, and so shaft harmonics from the 3rd to the 13th were retained in the TE signals. It was checked that the main effect of the crack was additive rather than multiplicative (modulation of the GM harmonics) so the signals were lowpass filtered just under half the GM frequency to enhance additive impulses from the crack, having components above the first two rotational harmonics, but removing modulation sidebands along with the GM harmonics.

Considering first the unfiltered results at low speed in Fig. 3(a), the increasing load gives a corresponding increase in the gearmesh component, but no change in a shaft speed component, which is likely due to a small eccentricity of the pinion. The TE for zero load could be taken as the GTE for this gear. The increasing GM component with load corresponds to the static deflection component of the STE.

For the equivalent results at 20 Hz, in Fig. 3(b), it is seen that the DTE is substantially different from the STE, at least with respect to the GM component. This can be explained by the fact that the GM frequency (540 Hz) is very close to a resonance of the system. This interpretation is also consistent with the fact that the increased GM component is dominated by the first harmonic, whereas that in Fig. 3(a) has many GM harmonics.

The filtered low speed results in Fig. 3(c) reveal the effect of the crack, at about 50 degrees along the scale, although the effect becomes less evident with increasing load. With this knowledge, it will be seen that the crack can also be detected in the unfiltered signal in 3(a), though only at the lowest load.

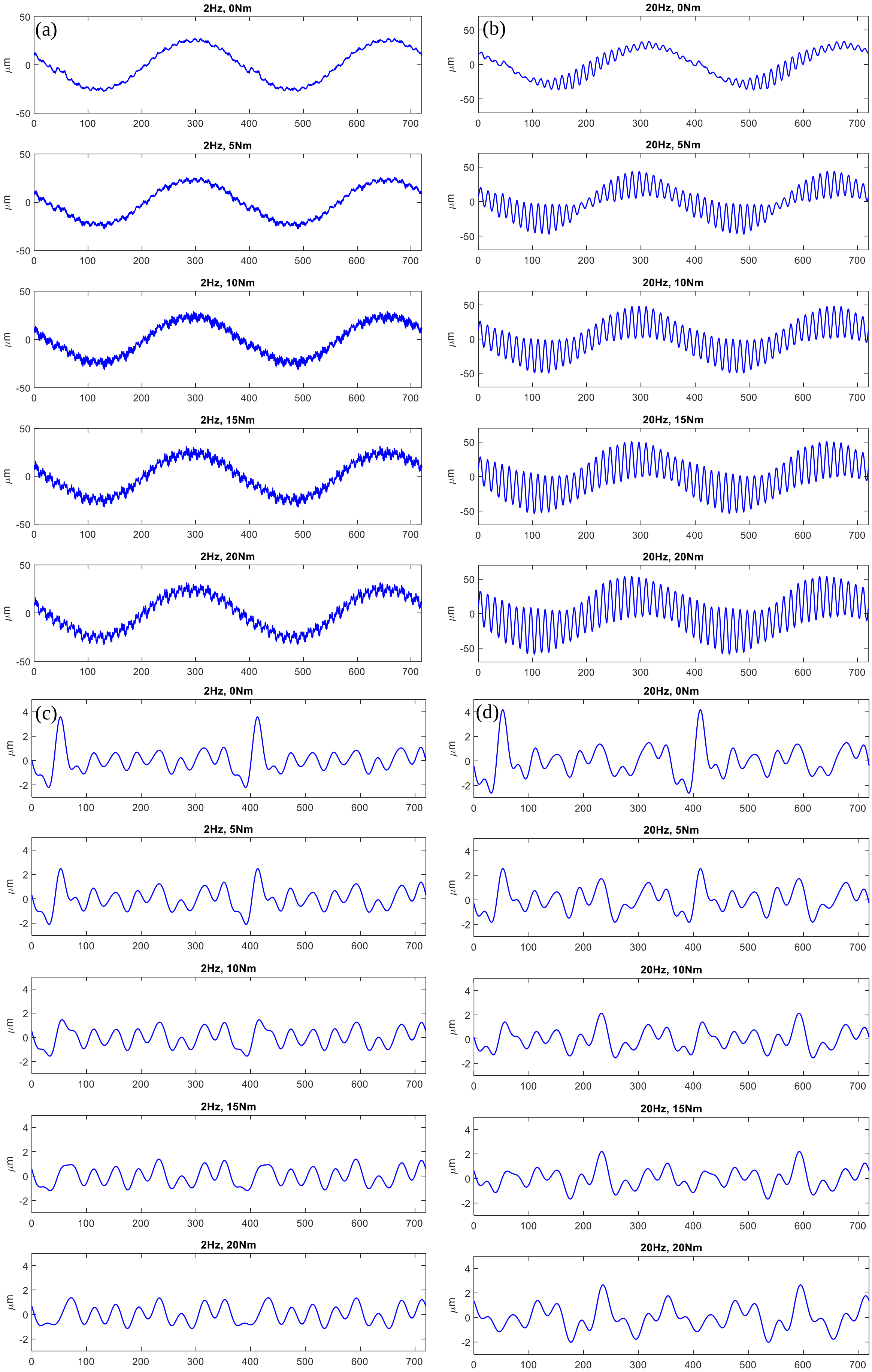

Figure 3: Comparison of original and filtered TE measurements
(a) Original TE, 2 Hz (b) Original TE, 20 Hz (c) Filtered TE, 2 Hz (d) Filtered TE, 20 Hz

The situation is very similar for the high-speed results in Fig. 3(d) (and 3(b)), and it is quite remarkable that once the effect of the resonance on the GM component is removed, the STE of Fig. 3(c) and DTE of 3(d) are very similar, at least for the lowest two loads. This illustrates one of the advantages of TE rather than vibration (including torsional vibration) as a diagnostic parameter, since the effects of operating conditions are greatly reduced.

The unexpected reduction in TE with increase in load gave rise to speculation as to the cause, and it was realised that it must be due to the fact that the “crack” has actually started slightly closed with respect to the undamaged gear, and the effect of increasing load is to counteract this with increasing tooth deflection under load. This is the opposite to what is expected to happen in the case of a genuine natural crack, where it has been demonstrated [5] that there is a tendency for the crack to be permanently open, in the unloaded condition, because of the plastic deformation at the crack tip which is an intrinsic part of crack development. The reason for the “crack” closure in this case is undoubtedly because of relief of residual stresses from heat treatment when the slot was machined, but this should never occur with real crack development, where STE due to loading would be in the same direction as the original GTE.

The change in TE as a result of tooth deflection is not easy to see, even from the filtered results in Fig. 3(c) and (d), but Figure 4(a) and (b) show a zoom of the differential TE in the vicinity of the crack. This represents the difference with respect to the curve at the highest load (20 Nm), but with reversed sign so as to show the increase of deflection with load. This is seen to be monotonic and close to linear. The corresponding linearised compliance can be derived from the deflection vs load curves in Fig. 4(c, d). These differ by only 33%, and indicate that it may be possible to estimate gearmesh stiffness from DTE as well as STE, even where measurements cannot be made at low speed.

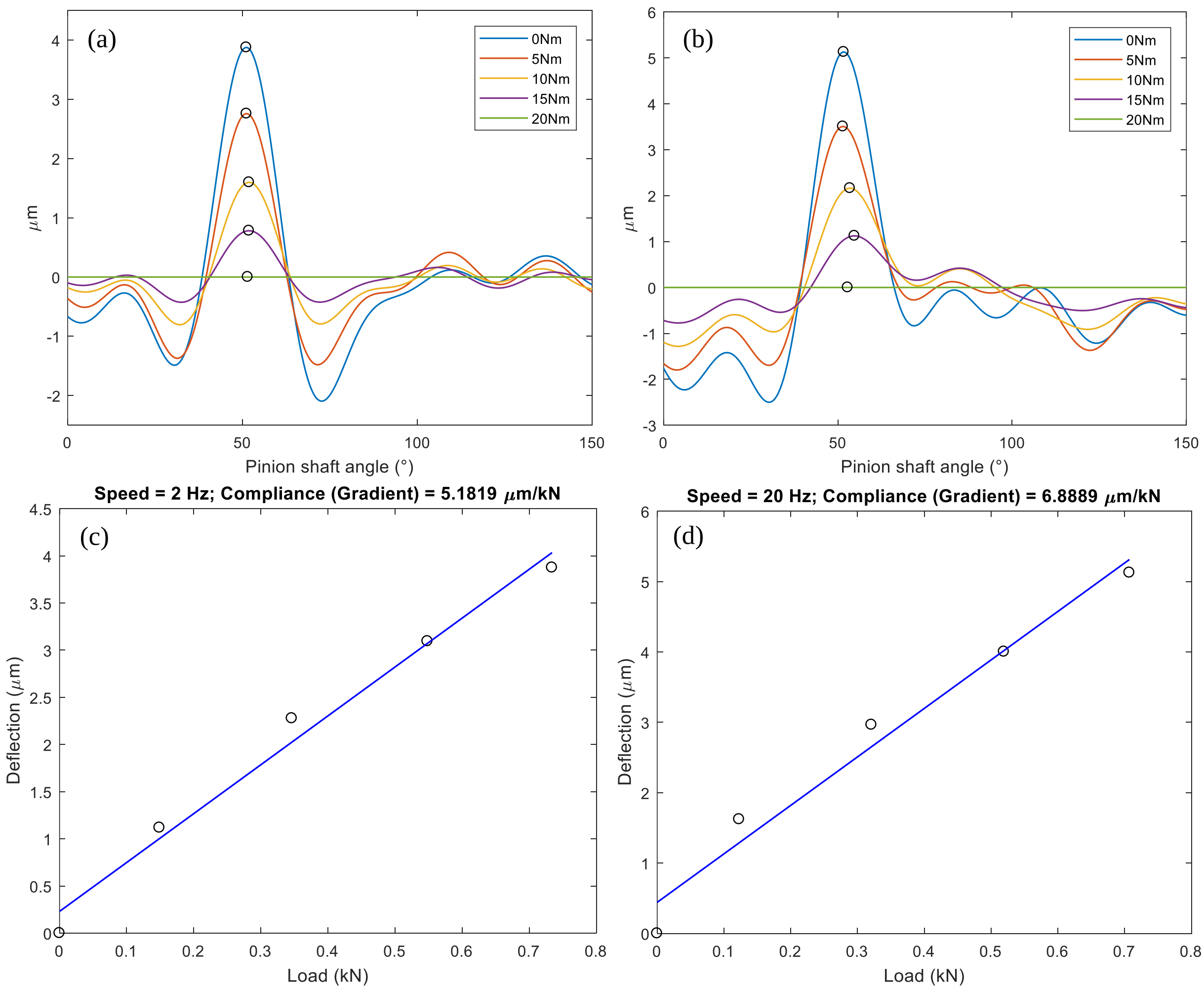

Figure 4: (a, b) Zoom on differential TE in vicinity of crack (c, d) corresponding compliance curves
(a, c) 2 Hz shaft speed (b, d) 20 Hz shaft speed

It is interesting to compare the (differential) compliance values in Fig. 4 with the typical value given for total stiffness by Smith in [6] as "A generally accepted figure for the mesh stiffness of normal teeth is 1.4 $\times 10^{10}$ N/m/m", which works out in this case to be 7×10^7 N/m, or 14 μm/kN in terms of compliance. This constant value (per unit facewidth) is based only on the bending stiffness component, and is independent of scale for a given shape of tooth since the stiffness varies directly with the cube of the depth, and inversely with the cube of the length. The values in Fig. 4 represent the differential compliance (additional deflection for the same load), which would be 5.18 and 6.89 μm/kN, respectively. In Ref. [7], an estimate is made of the change in stiffness of the toothmesh due to cracks of various sizes, using FEM and an improved simplified method, which agree. For their largest crack, which extends to 48.4% of the tooth thickness, and which has a sharp tip, the increase in compliance is 33% in the single tooth pair zone and 25% in the double tooth pair zone. Considering that the "crack" in the current results has a depth of 50%, and is actually a slot, it is likely the increase in compliance would be greater than those from [7], giving good agreement with the results from Fig. 4.

It is interesting to compare these TE results with those from response accelerations. Figure 5 shows synchronously averaged signals (over two rotation periods) at zero and 20 Nm load, and 2 and 20 Hz input shaft speed. Only the response at highest speed and highest load shows the tooth root crack. Although not shown here, even the responses at 20 Hz and 15 Nm did not show the crack. From Fig. 5(d) it appears that the effect of the crack is mainly multiplicative (local amplitude modulation) so it could be that the resonance near the GM frequency has also amplified the effect of the crack.

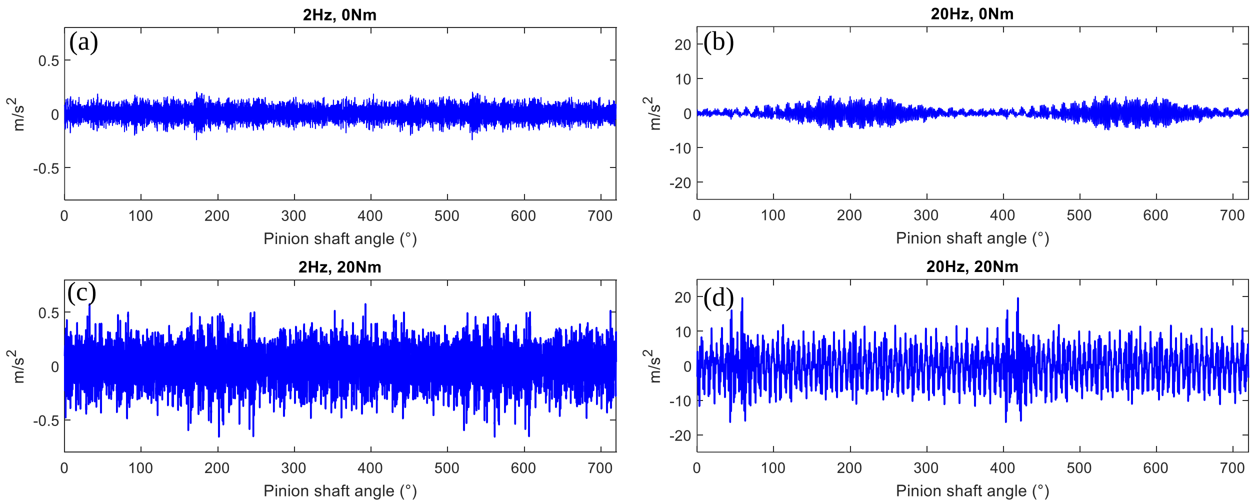

Figure 5: Synchronously averaged acceleration signals for two speeds and two loads
(a, b) 0 Nm (c, d) 20 Nm (a, c) 2 Hz (b, d) 20 Hz

It is quite possible that further signal processing could extract evidence of the crack from more of the response signals, but the main point with respect to this paper is that the TE and vibration responses give quite different information about a tooth root crack, with perhaps the main point being that it only excites a vibration response when teeth are deflected, and therefore not under zero load. The GTE, on the other hand, does show the crack at zero load, in this case because the "slot" had actually closed because of relief of residual stresses. However, in the case of normally developing cracks, they would be partially open because of plastic deformation at the crack tip, and would open further under load, this being detectable by measurement of STE and DTE, the latter at higher speeds, where it would not be possible to measure the GTE.

The fact that information was obtainable, from the measured TE, of toothmesh stiffness, even at higher speed where the GM frequency excited a resonance, emphasises the fact that the TE is measured right at the source, whereas vibration response measurements at different measurement points would all be different, and correspond to different (possibly time-varying) transmission paths.

4 Conclusion

This paper gives a number of examples of how measured gear TE can be useful in gear diagnostics, as an alternative, or supplement, to vibration measurements. It explains how GTE, STE and DTE can be measured if it is possible to run the machine at low speed and low load (GTE), low speed and high load (STE) and high speed and high load (DTE). An earlier paper demonstrated some of the characteristics for generalised distributed wear and pitting of the teeth, giving changes on tooth profiles, whereas the current paper shows a number of advantages, compared with vibration measurement, for the critical case of a tooth root crack. Of particular interest was that it was possible to obtain estimates of the change in toothmesh stiffness (actually compliance) due to the crack, and indirectly of the toothmesh stiffness itself. The latter would probably require comparison with simulations of the cracked tooth, for example with an FE model.

Potential advantages of using TE for gear diagnostics are:

1) The measurement is closer to the source, and less disturbed by transfer function effects than vibration responses, which not only vary considerably between different positions, but can also be time-varying.

2) It is easier to get a good correspondence with simulations, because the torsional parts of simulated systems are simpler, and affected by fewer resonances than lateral vibrations, so model updating should be simpler.

3) The measurement of GTE at different times during the life of a gearbox, as well as giving a more direct measurement of wear, will make possible the inclusion of more accurate versions of this parameter in simulation models, including those giving lateral vibrations as outputs.

The technique does require the mounting of accurate encoders on at least the input and output shafts of the gear transmission, but does not necessarily require them to be mounted on all shafts [3], which can be difficult for internal components. However, the inclusion of such encoders is already implemented in some machines, for operational purposes, and this is likely to increase with the wider implementation of the Internet of Things.

Acknowledgments

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References

[1] Sweeney, P.J. and Randall, R.B., *Gear Transmission Error Measurement using Phase Demodulation*. Proc. I.Mech.E., Part C, J.Mech.Eng.Sci. 210(C3), 1996, pp. 201-213.

[2] Randall, R.B., Vibration-based Condition Monitoring: Industrial, Aerospace and Automotive Applications. Wiley, Chichester, UK, 2011.

[3] Randall, R.B., Peng, D., Smith, W.A., *Using measured transmission error for diagnostics of gears*, SIRM conference, Copenhagen, February 2019.

[4] Chang, H., Borghesani, P., Smith, W.A., Peng, Z. *Application of surface replication combined with image analysis to investigate wear evolution on gear teeth - A case study*, Wear, 430-431, 2019, pp. 355-368.

[5] Mark, W.D., Reagor, C.P., Static-transmission-error vibratory-excitation contributions from plastically deformed gear teeth caused by tooth bending-fatigue damage, Mechanical Systems and Signal Processing, 21, 2007, pp. 885-905.

[6] Smith, J.D., Gear Noise and Vibration, 2nd Ed., Marcel Dekker Inc., NY, 2003.

[7] Mohammed, O.D., Rantatalo, M., Aidanpää, J.-O., *Improving mesh stiffness calculation of cracked gears for the purpose of vibration-based fault analysis*, Engineering Failure Analysis, 34, 2013. pp. 235–251.