Effects of Particle Swarm Optimization Algorithm Parameters for Structural Dynamic Monitoring of Cantilever Beam

Xiao-Lin LI[1,2], Roger SERRA[1,2] and Julien OLIVIER[1,3]

[1] Institut National des Sciences Appliquées Centre Val de Loire, Blois France,
[2] Laboratoire de Mécanique Gabriel Lamé - EA 7494, Blois, France
[3] Laboratoire d'Informatique Fondamentale et Appliquée de Tours, EA 6300, Blois, France

Abstract

Nowadays, particle swarm optimization (PSO) algorithm has become a widespread optimization method. However, it is well known that its main parameters (inertia weight, two learning factors, velocity constraint and population size) have a critical effect on its performance. Currently the effects of PSO parameters on structural health monitoring have not been comprehensively studied. Therefore, in this paper, the PSO algorithm is used for damage detection assessment of a cantilever beam, and the simulation results are used to analyze the effects of PSO parameters. There are five levels for each parameter in our experiment, mean fitness value and success rate for each level are used as criteria to measure the convergence and stability of the PSO algorithm. Considering the effect of population size on CPU time, a trade-off strategy is presented to further determine the selection of population size.

1 Introduction

Over the last few years, there have been increasing demands to develop structural dynamic monitoring system over different kinds of aerospace, mechanical and civil engineering structures because of the huge economic and life-safety benefits. Vibration testing is the widest used method for structural damage detection[1, 2]. The main idea behind damage detection techniques based on structural dynamic changes is the fact that the modal parameters of a structure are functions of the physical parameters (such as mass, stiffness and damping) thus the existence of damage leads to changes in the modal properties of the structure. The inverse method for damage detection using vibration data and solving by optimization algorithms have received extensive attention in recent years. The usual approach is to minimize an objective function, which is defined in terms of discrepancies between the predicted model parameters and the initial model parameters. Using classic optimization methods to solve it often meet some difficulties. However, the particle swarm optimization (PSO) algorithm can be used on complicated optimization problems that cannot be expressed explicitly. PSO algorithm is one of the newest intelligent method, this parallel evolutionary computation technique was developed by Kennedy and Eberhart in 1995[3].The basic idea comes from the study of group behaviors such as predation of birds. PSO algorithm was first used for function optimization and neural network training[3]. Since the algorithm has many advantages such as comparative simplicity, easy to implement and few parameters to be adjusted, PSO has found its application in many complex engineering optimization problems, including structural damage detection of beam structure[4, 5].

It is well known that in various optimization methods, parameters is one of the key factors which have a great effect on the performance. For different kinds of optimization problems, the matching and cooperation modes between parameters are different. Even for the same type of optimization problem, if problem scales are different, parameter selections are not completely the same. Although PSO algorithm has few parameters to adjust, how to determine them is also an important problem. However, currently the effects of PSO parameters on structural damage detection have not been comprehensively studied. In this paper, the damage detection of a cantilever beam by PSO algorithm for two damage patterns are simulated, and the experimental results are analyzed to study the effects of PSO parameters. There are five levels for each parameter in our experiment, the mean fitness value and success rate for each level are used as criteria to measure the convergence and stability of the PSO algorithm. Considering the effect of population size on CPU time, a *Ratio* compromise strategy is proposed to further determine the selection of population size.

2 Problem formulation

2.1 Structural dynamic finite element formulation

The governing equation for an Euler-Bernoulli beam with negligible damping is given by:

$$\frac{\partial^2}{\partial x^2}\left[EI(x)\frac{\partial^2\omega(x,t)}{\partial x^2}\right]+m(x)\frac{\partial^2\omega(x,t)}{\partial t^2}=f(x,t) \tag{1}$$

where $EI(x)$ denotes the flexural stiffness, $m(x)$ denotes the mass per unit length of the beam, $w(x,t)$ represents the transverse displacement of the beam. The beam is discretized into a number of elements, with displacement and slope as nodal degrees of freedom and cubic interpolation function.For an n-degree of freedom system of uniform beam, the stiffness matrix and the consistence mass matrix are given in [6], respectively.

The characteristics of the beam are given in the table below. The total mass is 3.237 kg.

Young modulus E (Pa)	Poisson ratio ν	Density ρ $(Kg.m^{-3})$	Length L (m)	Width b (m)	Thickness h (m)
2e+11	0.33	7850	1	5.3e-3	2.5e-2

Table 1: Beam properties

For a properly modeled structure, the structural dynamic eigenvalue equation is given by:

$$(K-\lambda M)\Phi=0 \tag{2}$$

where K and M are the global stiffness and mass matrices respectively, and λ and Φ represent the natural frequency and vibration mode shape vectors. It can be assumed that when a structural modification occurs, the local stiffness of the structure changes whereas the change in mass may be neglected. Hence, the equation (2) could be rewritten for a modified system as:

$$(K_d-\lambda_d M)\Phi_d=0 \tag{3}$$

where K_d and M are the global modified stiffness and mass matrices respectively, and λ_d and Φ_d represent the new natural frequency and vibration mode shape vectors for the modified structure respectively. In many studies on structural health monitoring, the structural modification has been simulated by decreasing one of the local element 's stiffness parameters like a inertia moment I, cross sectional area S or elasticity (Young) modulus E. In this work, the structural modification of each element was simulated using Young modulus reduction factor (also called stiffness reduction factor) x as a scalar variable between $[0,1]$ where zero value corresponds to no modification and a value near to one corresponds to failure condition introduced as follows[5]:

$$x_i=\frac{E-E_i}{E} \tag{4}$$

where E is the initial Young modulus and E_i is the reduced Young modulus of the i^{th} element. In this case, the stiffness matrix K will be modified as:

$$K_d=\sum_i(1-x_i)K_i \tag{5}$$

2.2 Optimization problem

Equation (3) forms the basis of the structural modification detection method through an inverse procedure giving the new vibration natural frequencies and the mode shapes. As the structural modification causes change in vibration natural frequencies and which are easier to measure than mode shapes (limited number of accelerometers) and the error associated is comparatively less. Hence, they are used as structural modification indicators in this study. PSO algorithm is used to search a particular stiffness reduction factor x so that the predicted numerical natural frequencies exactly match with the initial natural frequencies. The initial numerical model of the structure is generally considered for the optimization. When the exact match between modified and initial natural frequencies is observed, the value of stiffness reduction factor represents the actual modification location and amount. The usual approach to solve the inverse problem of structural dynamic monitoring involves minimization of the fitness function (or objective function) which is defined in terms of discrepancies between the predicted natural frequencies and the initial natural frequencies. In this study, the fitness function can be presented just like in literature [7]:

$$F = \sum_{s=1}^{n} \left(\frac{(f_s^r)^2 - (f_s^p)^2}{(f_s^r)^2} \right)^2 \tag{6}$$

where f_s^r and f_s^p are the initial and predicted natural frequencies respectively. n is the number of input response parameters chosen (natural frequencies) and for this study is taken as five.

2.3 Particle swarm optimization algorithm

The particle swarm optimization technique is a population based stochastic technique in nature (bio-inspired) so-called evolutionary computational model which is based on swarm intelligence. PSO is developed by Kennedy and Elberhart [3] and primarily used to tackle continuous optimization problems. The system is initialized firstly in a set of randomly generated potential solutions, and then performs the search for the optimum one iteratively by swarms following the best particle. Compared to others evolutionary algorithms, PSO has much more profound intelligence background and could be performed more easily. Based on its advantages, the PSO is suitable for engineering applications, in the fields of evolutionary computing, optimization and many others.

As suggested in literature [8], a fully connected topology is elected as PSO algorithm topology. Set for the D-dimensional search space, mth particles compose a population $\{X_1, X_2, \ldots, X_m\} \subset R^n$, and the ith particle position is $X_i = (x_{i1}, x_{i2}, \ldots, x_{iD})^T$, the velocity of the particle X_i can be represented by another D-dimensional vector $V_i = (v_{i1}, v_{i2}, \ldots, v_{iD})^T$. The best position previously visited of the particle X_i is denoted as $P_i = (p_{i1}, p_{i2}, \ldots, p_{iD})^T$, and the best position among all particles in the population is $P_g = (p_{i1}, p_{i2}, \ldots, p_{iD})^T$. Each particle adjusts its position dynamically according to the principle of following the current optimal particle, the particle X_i updates its speed and position according to (7) and (8).

$$v_{id}^{t+1} = \omega v_{id}^t + c_1 r_1 (p_{id}^t - x_{id}^t) + c_2 r_2 (p_{gd}^t - x_{id}^t) \tag{7}$$

$$x_{id}^{t+1} = x_{id}^t + v_{id}^{t+1} \tag{8}$$

where t is iteration time, d represents the dimension of the particle, $d = 1, 2, \ldots, D$, i represents the number of the particle, $i = 1, 2, \ldots, m$. r_1, r_2 are random between 0 and 1, ω is the inertia weight, c_1 and c_2 are the learning factors to adjust each iteration step length.

many literatures has emphasized the importance of ω, the linear decreasing inertia weight ω^t has been widely used[9], which is defined as follow:

$$\omega^t = \omega_{max} - \frac{\omega_{max} - \omega_{min}}{t_{max}} \times t \tag{9}$$

where t_{max} is the maximum number of iteration. In this strategy, ω^t changes with iteration. At beginning, the value of ω^t is ω^{max} , and ω^t decrease during the execution of the algorithm. At the end the value of ω^t is ω_{min}. ω_{max} and ω_{min} are set to 0.9 and 0.4 respectively.

3 Analysis of parameters on the algorithm performance

In the experiment, a steel cantilever beam is considered for structural damage detection. Figure 1 shows the sketch of the beam with element number using in the finite element simulations, 30 equal Euler-Bernoulli beam elements are chosen for finite element modeling.

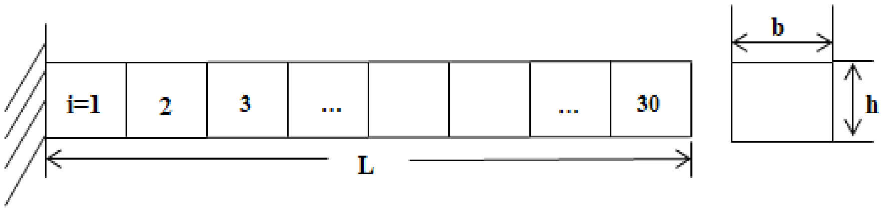

Figure 1: The Euler-Bernoulli cantilever beam model

PSO algorithm has some important parameters, such as population size (m), inertia weight (ω), two learning factors (c_1, c_2) and maximum velocity $(Vmax)$. The effects of PSO parameters are analyzed by simulating structural damage detection of a cantilever beam. 200 iterations and 1000 runs are set for two damage patterns in Table 2.

Damage Pattern I		Damage Pattern II	
Element	Damage(%)	Element	Damage(%)
5	10	5	10
		7	10

Table 2: Simulated damage patterns in cantilever beam

Table 3 shows the parameter setting. The value of $Vmax$ is set to $Vmax = \gamma Xmax$ and $\gamma \in (0, 1]$, $Xmax$ denotes the dynamic range of the variable on each dimension. In each test, there are five levels for each parameter, mean fitness value is equal to the average value of fitness function F, success rate is equal to the ratio of the number of successful runs to total number of runs, which are all considered as criteria for parameter performance measurement. When fitness function provides smaller mean fitness value it shows better convergence performance of the algorithm. The higher success rate provides the stronger stability of the algorithm. The convergence and stability for each parameter at five levels are compared.

Table 3: parameter setting for five levels

level	population size m	inertia weight ω	learning factors (c_1, c_2)	maximum velocity $Vmax$
1	20	0.25	$(0, 4)$	$0.2Xmax$
2	40	0.5	$(1, 3)$	$0.4Xmax$
3	60	0.75	$(2, 2)$	$0.6Xmax$
4	80	1.0	$(3, 1)$	$0.8Xmax$
5	100	ω^t	$(4, 0)$	$Xmax$

3.1 Effect of population size m

The choice of population size is related to the complexity of the problem. As the complexity of the problem increases, the population size also grows. The five levels of population size are given in Table 3, the other parameters are chosen as $\omega = \omega^t, c_1 = c_2 = 2, Vmax = Xmax$ [9]. Mean fitness value and success rate for two damage patterns are shown in Figure 2. It is clearly that along with the increase of population size m, the convergence and stability of the PSO algorithm is becoming stronger and stronger.

However, for a given problem, the parameters that affect CPU time are mainly finite element number, population size, maximum iterations and number of PSO runs, they are given except for population size. Therefore, when the effect of population size are analyzed, CPU time needs to be taken into consideration. The larger the population size represents the longer CPU time. Then, a trade-off strategy (10) is proposed as a criterion to further determine the population size m. Obviously, a larger $Ratio$ means a better performance with selected population size. Thus, it can be seen in Figure 3 that for two damage patterns,the optimal choices are $m = 20$

and $m = 60$,respectively.

$$Ratio = \frac{\text{success rate}}{\text{population size}} \tag{10}$$

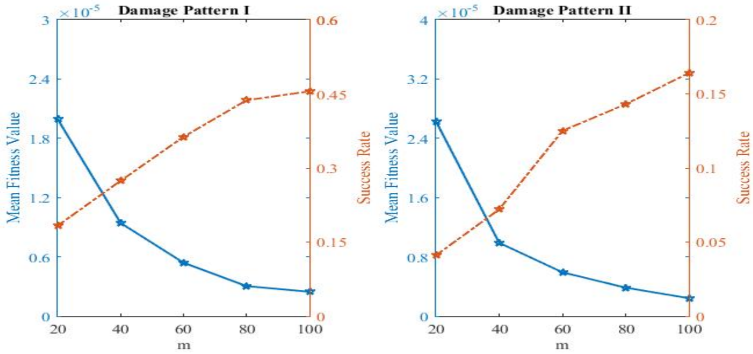

Figure 2: Effect of population size on PSO algorithm for two damage patterns

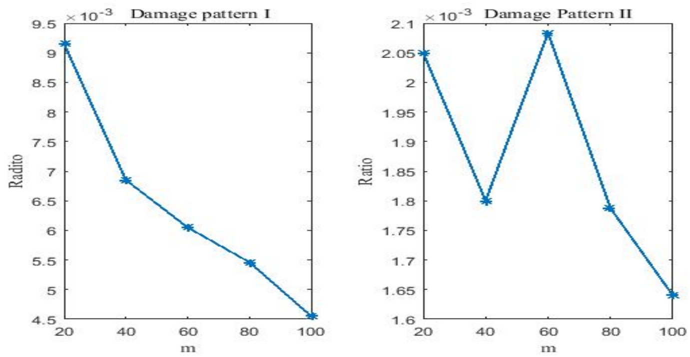

Figure 3: Selection of population size for two damage patterns

3.2 Effect of inertia weight ω

The inertia weight ω affects the particle's global and local search ability. The stochastic process theory in [10] shows that the range of ω is $[0,1]$. From the above,the best setting for population size are: $m = 20$ for damage pattern I, $m = 60$ for damage pattern II. And $c_1 = c_2 = 2, Vmax = Xmax$. In order to examine the balance between global and local exploration, the five levels of inertia weight are compared, and the simulation results are shown in Figure 4.

When ω is small, PSO algorithm hardly converges and the success rate is low. Along with the increase of ω, PSO algorithm has a better convergence and stability. Although a better convergence and stability can also been obtained when ω takes linear-decreasing strategy, the optimal choice is $\omega = 1$ which is different from the general linear-decreasing strategy.

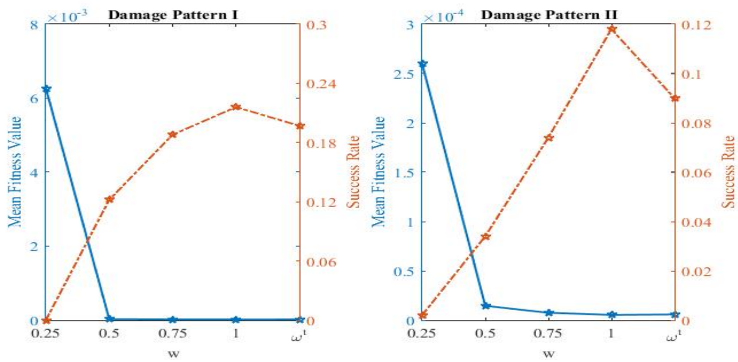

Figure 4: Effect of inertia weight on PSO algorithm for two damage patterns

3.3 Effect of learning factors c_1, c_2

A general rule for setting the two learning factors is $c_1 + c_2 <= 4$, there is a close relationship between c_1 and c_2. Therefore, the values of the two learning factors are considered simultaneously. Considering the simplicity of the experiment, the following relation between c_1 and c_2 will be used:$c_1 + c_2 = 4$, the five levels of the learning factors are compared with $\omega = 1, Vmax = Xmax$, $m = 20$ for damage pattern I, $m = 60$ for Damage Pattern II.

From figure 5, it can be seen that $(c_1, c_2) = (3,1)$ is the optimal choice for the convergence and stability of PSO algorithm. That means, for structural damage detection of the cantilever beam, the algorithm shows a good performance when the population put much more attracted to the best location found by itself.

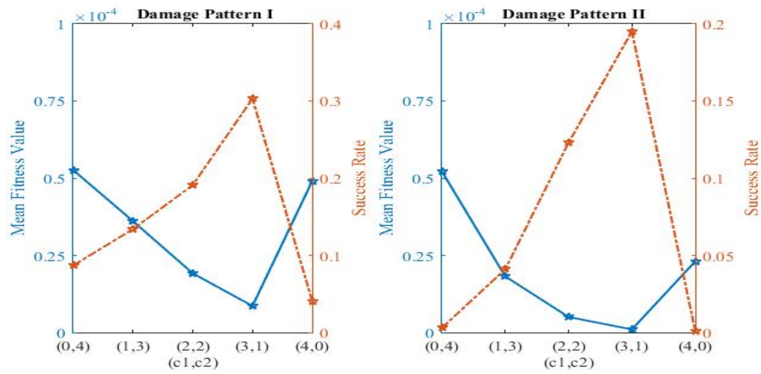

Figure 5: Effect of leaning factors on PSO algorithm for two damage patterns

3.4 Effect of maximum velocity $Vmax$

The velocity of the particles can be limited to $[-Vmax, Vmax]$ by a maximum velocity, which acts as a constraint to control the global exploration capability of the population, It is clearly that $\omega = 1, c_1 = 3, c_2 = 1$,$m = 20$ for damage pattern I,$m = 60$ for damage pattern II are optimal choices for structural damage detection. Then, the convergence and stability of $Vmax$ for the five levels are compared.

The simulation results for mean fitness value and success rate are shown in Figure 6. Along with the increase of the maximum velocity, mean fitness value is decreasing and success rate is increasing, which means the convergence and stability of the algorithm is becoming stronger and stronger. Usually set $Vmax$ as a constant, $Vmax = Xmax$ is the best choice for structural damage detection of the cantilever beam.

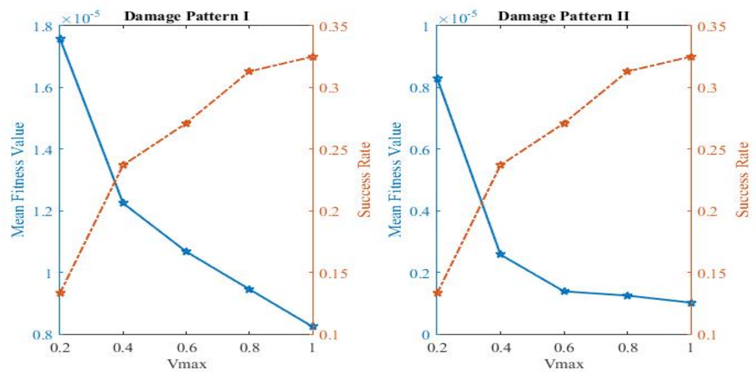

Figure 6: Effect of maximum velocity on PSO algorithm for two damage patterns

4 Validation

From the above paper, the optimal parameter configuration for damage pattern I is $(m, \omega, c_1, c_2, Vmax) = (20, 1, 3, 1, Xmax)$, at this time, mean fitness value $= 8.60e-06$, success rate $= 0.36$. For damage pattern two is $(m, \omega, c_1, c_2, Vmax) = (60, 1, 3, 1, Xmax)$, the corresponding mean fitness value and success rate are $9.78e-07$

and 0.18, respectively. Under the optimal configuration, PSO shows a better convergence and stability than other configurations used in our experiment by using relatively low time costs. Therefore, the effectiveness is verified by simulating structural damage detection of cantilever beam.

5 Conclusion

In this paper, first of all, the five levels for each parameter are designed to perform damage detection of cantilever beam. Then mean fitness value and success rate obtained from simulation results are used as criteria to evaluate the convergence and stability of the algorithm. Considering CPU time, the *Ratio* strategy is proposed to further determine the selection of population size. A parameter guideline are given for structural damage detection of cantilever beam.

References

[1] S. W. Doebling, C. R. Farrar and M. B. Prime, *A Summary Review of Vibration-Based Damage Identification Methods*, The Shock and Vibration Digest, 1998.

[2] Y.J. Yan, L. Cheng, Z.Y. Wu and L.H. Yam, *Development in Vibration-based Structural Damage Detection Technique*, Mechanical Systems and Signal Processing, 2007.

[3] J. Kennedy and R.C. Eberhart, *Particle swarm optimization*, Proceedings of IEEE international conference on neural networks, 1995.

[4] B. Nanda, D. Maity and D. K. Maiti,*Vibration Based Structural Damage Detection Technique using Particle Swarm Optimization with Incremental Swarm Size*, International Journal of Aeronautical and Space Sciences, 2012.

[5] G. Ghodrati Amiri, A. Zare Hosseinzadeh and S.A. Seyed Razzaghi, *Generalized Flexibility-based Model Updating Approach via Democratic Swarm Optimization Algorithm for Structural Damage Prognosis*, International Journal of Optimization in Civil Engineering, 2015.

[6] J. Lee, *Identification of Multiple Cracks in a Beam Using Natural Frequencies*, Journal of Sound and Vibration, 2009.

[7] L. Rubio, J. Fernández-Sáez and A. Morassi. *Identification of Two Cracks with Different Severity in Beams and Rods from Minimal Frequency Data*, Journal of Vibration and Control, 2014.

[8] A. Carlisle and G. Dozier, *An Off-the-shelf PSO Proceedings of the Workshop on Particle Swarm Optimization*,Proceeding of the 2001 Workshop on Particle Swarm Optimization, Indianapolis IN, 2001.

[9] R. Perera, S-E Fang and A Ruiz, *Application of Particle Swarm Optimization and Genetic Algorithms to Multiobjective Damage Identification Inverse Problems with Modelling Errors*, Information Processing Letters, 2007.

[10] M. Jiang, Y. P. Luo and S. Y. Yang, *Stochastic Convergence Analysis and Parameter Selection of the Standard Particle Swarm Optimization Algorithm*, Information Processing Letters, 2007.