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# Bifurcation Tracking and sub-harmonic isola detection in nonlinear mechanical systems

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## Abstract

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Intermittent contacts are commonplace in numerous mechanical engineering applications. Since the resulting localized forces are strongly non-linear, multiple dynamical regimes may co-exist for a given set of conditions, and small perturbations then suffice to induce a change from a desirable to a potentially unfavourable state. For this reason, prediction of global dynamics is critical, especially in regard to the systems' parameters. Numerical *bifurcation tracking* offers a means to efficiently compute stability boundaries, as done by Xie et al. [1] for systems under forced vibration. In this approach, constraints are appended to the equations of motion, and the solution of the *extended systems* thus constructed gives the loci of bifurcation points for a varying parameter. Even isolated resonance curves, which are usually difficult to detect, have been found through this technique, see e.g. Kuether et al. [2]. However, this has only been done for the case when both the regimes on the isolated and main resonance branches share a common fundamental frequency, whereas it is known that *sub-harmonic* isolated resonances are also possible for vibrating systems undergoing asymmetric contacts [3].

In the present contribution, we first propose an extended system characterizing period doubling bifurcations through the *harmonic balance method* (HBM), then proceed to track these bifurcations. We show that, given a judicious choice of tracking parameter, extremal points on the ensuing stability boundaries correspond to the birth of isolated sub-harmonic resonances. Moreover, we further characterize and track these points with respect to a second parameter. This is interesting from a design viewpoint, since it provides a way to avoid these regimes altogether. Afterwards, we present an example application on an academic system, where an "asymmetry parameter" controls the existence of sub-harmonic isolas. Finally, numerical results are confronted with experimental measurements, which validate the methods proposed herein.

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