# Localization and quantification of damage by frequency-based Method: Numerical application on bending vibration beam

Anurag DUBEY[1], Vivien DENIS[1], Roger SERRA[1]

[1]INSA Centre Val de Loire, Laboratoire de Mécanique Gabriel Lamé - EA 7494, Blois, France
anurag.dubey@insa-cvl.fr

## Abstract

The sudden growth of damages can cause catastrophic failure of structures or mechanisms that lead to unplanned shutdowns of machines and production lines. If a damage remains undetected and reaches a critical size, sudden collapses and failures can happen. To overcome these problems, it is essential to detect these damages before they reach their critical state. The presence of damages can alter the structure which reduces the bending stiffness and modify the modal parameters and the natural frequencies. One of the most suitable monitoring methods to define the presence of damage and assess the structure is vibration based structure health monitoring (VBSHM). The objective of the work is to localize and quantify the damages with the consideration of eigenfrequencies of healthy and tested structures. Hence, a methodology for damage identification in structure using frequency shift coefficient (FSC) is presented. Numerical finite element models (2D and 3D) are performed and correlated to obtain a damage library for the cantilever beam structure. Based on the cost function, Young's modulus of 2D and 3D models are iteratively updated to closely match the frequencies of the reference beam. The approach also quantified geometry damage with vibration measurements on cantilever beams, which is related to an equivalent bending stiffness reduction by the use of FSC. The effect of severity of the damage is considered. Finally, the result is validated numerically through the identification of geometry damage.

## 1 Introduction

Damages or cracks are inevitable in aerospace, aeronautical, mechanical and civil structures during their service life. Any changes in the structures such as material, physical or geometrical properties which affects their performance are considered as damages. The study of damages is an important perspective in order to ensure safety or to avoid any serious losses. Sudden occurrence of damages in the structure can cause catastrophic failure and reduction in load carrying capacity. However, it is necessary to improve the durability and reliability of structure as expressed in the design and maintenance specifications. The presence of the damage makes local stiffness vary in the structure and it also affects the mechanical behavior and performance of the structure. However, preventing the formation of damages is almost impossible as they propagate along the structure due to fluctuating stress or fatigue conditions. If these cracks remain undetected and reach a critical size sudden collapse can happen. Indeed, damage identification has significant life safety implications.

Structure Health Monitoring (SHM) is an efficient way for the diagnosis of the constituent's materials or structures. SHM involves the integration of sensors, data transmission, computational techniques, and processing ability to respond the behavior of a structure. Consequently, it aims to provide maintenance services throughout the life of the structure. Nowadays, structural damages are identified by Non-Destructive Testing (radiographic, ultrasonic testing, X-ray, eddy-current etc.) [1]. Vibration based structural health monitoring (VBSHM) is one of these categories based on the fact that a loss of stiffness caused by damages affects the dynamic response of the structure. VBSHM consists of five levels (existence, location, type, extent and prognosis) [2] which are efficient and widely accepted because of their ability to monitor and detect damage from global testing of the structure.

Many researchers from the last few decades, natural frequencies of a damaged structure are found as an identification parameter for both damage location and size. The first study developed by Cawley and Adams [3] depends on the shift of more than one frequency that could yield the location of the damage. In a review of the literature, Salawu [4] found that the natural frequencies are a sensitive indicator to detect the damage in the structure. The important technique is analyzing the changes (shifts) in natural frequencies in a structure with and without damage. Hilmy et al. [5] have presented frequency shifting as a function of damage evolution for a plate structure. The method proves shifting of the natural frequency is greater at higher frequency values and determines the location of the void damage. Messina et al. has proposed Damage Location Assurance Criterion (DLAC) [6] and after extended to Multiple Damage Location Assurance Criterion (MDLAC) [7] to measure the frequency variation due to damage between experimental and numerical values correlation. More recently, a method proposed by Serra et al. [8] demonstrates a correlation of 2D and 3D FE models to identify the typical damages (like hole, crack, notch) based on numerical and experimental study. Masoumi and Ashory [9] presented numerical and experimental studies to localize cracks.

In this paper, an approach for damage identification by using the frequency-shift coefficient is proposed. This method was first introduced by Silva and Gomes [10] for solving the damage detection problem. The method requires numerical models as a function of damage position and size for the frequency shift. First, vibration based strategy is used with detection, localization and classification (Size/Severity/Geometry) of damages. The study is followed by simulating a beam in commercial software (COMSOL, MATLAB) as a numerical case and 2D and 3D FE models are correlated to obtain geometry damage properties (size, location and severity...). Finally, numerical example is validated in order to localize and quantify geometry damage.

## 2 Cantilever beam bending vibration background

The eigenvalue problems and the analytic formulas concerning the modal parameters of a cantilever beam were described by the partial differential equation of the linear model with viscous damping as:

$$M(x)\frac{\partial^2 v(x,t)}{\partial t^2} + C(x)\frac{\partial v(x,t)}{\partial t} + \frac{\partial^2}{\partial x^2}\left(EI(x)\frac{\partial^2 v(x,t)}{\partial x^2}\right) = F(x,t) \tag{1}$$

where $v(x,t)$ is the transverse deflection, $M(x)$ is the mass per unit length, $C(x)$ is the damping coefficient, $EI(x)$ is the bending stiffness and $F(x,t)$ is the external force per unit length of the beam. The equation of the motion for dynamic systems are easily obtained from Newton's second law. This gives an equation for each degree of freedom within the system. When discretized, the equation of the motion may take the following matrix form:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{F\} \tag{2}$$

where $[M]$ is the mass matrix for the system, $[C]$ is the damping matrix and $[K]$ is stiffness matrix, $\{\ddot{X}\}$, $\{\dot{X}\}$, $\{X\}$ are vectors containing acceleration, velocity and displacement in all degree of freedom of the model; and $\{F\}$ contains external forces actuating in the system. If we assume free motion and negligible damping, one possible solution for the equation is:

$$\{x\}_i = \{y\}_i \sin(\omega_i t - \theta_i) \tag{3}$$

$y_i$ are the amplitudes for each mode shape, $\omega_i$ are the natural pulsations (in rad.s$^{-1}$) of vibration for each mode shape and $\theta_i$ are phase angles. The natural frequencies (in Hz) are given by $f_i = \frac{\omega_i}{2\pi}$. The following equation is obtained for the healthy case:

$$([K] - \omega_i^2[M])y_i = 0 \tag{4}$$

Damage to the structure changes its dynamic response. Therefore, natural frequencies and natural modes are changed. The equation of a damaged case can be expressed as:

$$([K] - (\omega_i^2)^*[M])y_i^* = 0 \tag{5}$$

### 2.1 2D Finite element model

The studied model is a cantilever beam, which has two degrees of freedom, a vertical translation $y$ and a rotation $\theta_z$. As can be seen in (Figure 1) this beam is divided into equal size of $N$ elements and $N+1$ nodes.

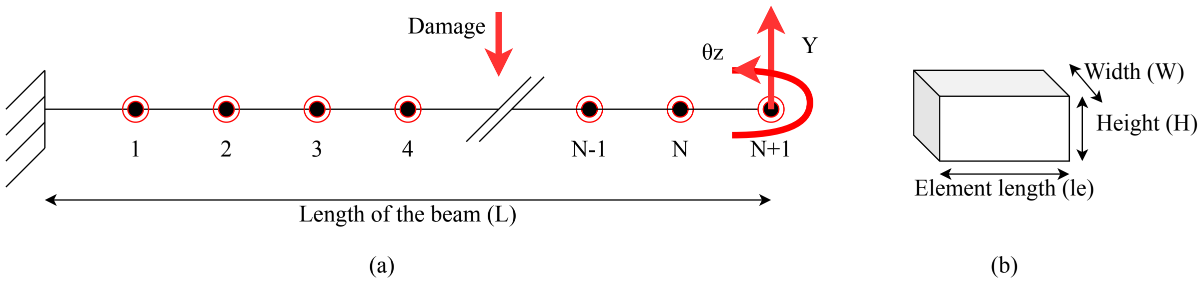


**Figure 1 –** Cantilever beam model. (a) 2D Finite element model; (b) element properties

In 2D FE Model, the damage is represented by an elemental stiffness reduction coefficient $\alpha_i$ which is the ratio of the stiffness reduction to the initial stiffness. The stiffness matrix of damaged beam is defined as a sum of elemental matrices multiplied by reduction coefficient by the following equation:

$$[K_d] = \sum_{i=1}^{N}(1-\alpha_i)[K_i] \tag{6}$$

where $K_d$ is global stiffness matrix for damaged beam, $K_i$ is elemental stiffness matrix, $N$ is number of elements, and $\alpha_i$ is a reduction coefficient , which varies from 0 to 1 for the damaged structure. The value of $\alpha_i$ = 0 indicates a healthy structure.

### 2.2 3D Finite element model

Simulation of damaged beam structure is performed using COMSOL multiphysics software. The damage model is built and the mesh is 3D tetrahedron element. The number of mesh is controlled by the software and depends on the shape of the structure, thus it changes with the size of the crack. A high meshing density is applied near the damaged area mainly to have the behavior correctly modeled.

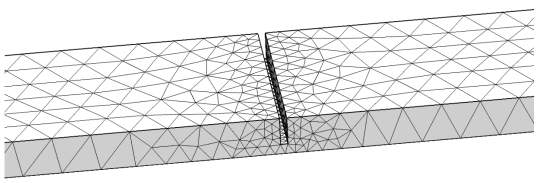

**Figure 2 –** 3D finite element mesh of the beam with damage

Geometry case (rectangular) is studied in order to quantify the severity of the damage. Figure 2 shows the meshed beam zoomed near the damaged area and width of the crack is set to 0.5 mm while the height is a parameter. The sensitivity of the 3D model is determined by mesh size. As the mesh is finer the model is more sensitive but computing cost is higher.

## 3 Frequency shift coefficient based strategy

The first type of modal method for damage detection relied on changes in dynamic properties of the structure and particularly natural frequencies. Any changes in the properties of the structure, such as reduction in stiffness will cause changes in the natural frequencies. One of the important advantages of natural frequency is that it can be quickly and easily conducted when measurements required. Classical measurements procedure can be used for the determination of experimental resonant frequencies. In this context, the frequency shift criterion is first presented by Silva and Gomes [10] for damage identification problems. The technique requires experimental measurements or numerical solution for the frequency shifts as a function of size and position of damage The frequency shift coefficient (FSC) is defined as:

$$FSC = \text{argmin}\left(\sqrt{\frac{1}{m}|\sum_{i=1}^{m}\left(\frac{(R_i)_X-(R_i)_A}{(R_i)_X}\right)|}\right) \quad \text{and} \quad R_i = \frac{f_i^u}{f_i^h}, \tag{7}$$

where $m$ is the total number of modes, $X$ refers to the tested case, $A$ refers to the reference case, $f_i^u$ is the unknown beam frequencies, $f_i^h$ is healthy beam frequencies and $i$ is denotes modes indices.

It is well known that the presence of damages modifies dynamic parameters and behavior of the structure. The location, classification and size of damages in the structure are identified by changes in the vibration parameters. At first, a set of reference state frequencies are identified. Numerical correlation of 2D and 3D FE models is performed to fit the frequencies with the references. Then, in the 3D FE model, the damage was materialized as a geometrical discontinuity of rectangular considering the position, type, size, geometry of the damage. At the same time, the damage was materialized as a local reduction of bending stiffness in an element for the 2D FE model. Finally, numerical correlation result will specify the position, size, depth and geometry of the damage from the damage library.

## 4 Numerical rectangular geometry damage identification

The numerical simulation test is performed to verify the efficiency of the proposed VBSHM strategy. A cantilever steel beam was taken into consideration for the numerical test and beam properties are given below in Table 1. A beam 2D FE model was divided into equal size of 100 elements and each element size is 10 mm.

| **Beam Properties** | **Value** |
|---|---|
| Length (L) | 1000 mm |
| Width (W) | 24.9 mm |
| Height(H) | 5.3 mm |
| Young's modulus (E) | 210 GPa |
| Mass density ($\rho$) | 7850 kg/m$^3$ |
| Poisson's ratio (v) | 0.33 |

**Table 1 –** Beam dimensions and properties

However, 2D-3D model correlation is done for the damage geometry (rectangular) by using software COMSOL with MATLAB. The modal responses of the structure were generated using FE models before and after the damaged case. The first seven modes are retained. The criterion is employed as a tool for identification of the damage by measuring frequencies. The final goal of correlation is to localize and quantify the severity of geometry damage that can link to the percentage reduction in stiffness of a beam structure.

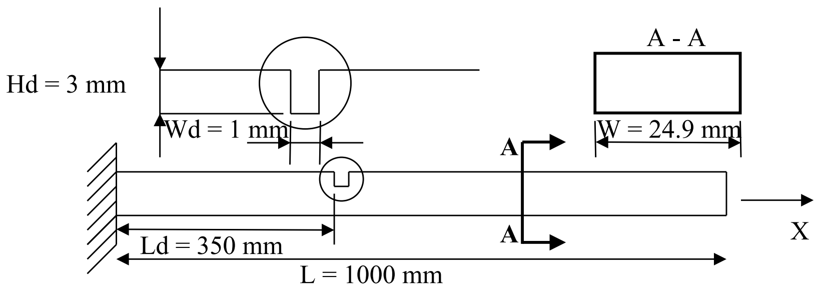


**Figure 3 –** 2-D clamped free beam plan with rectangular damage

A detailed 2D beam view as shown in Figure 3. The damage case is tested for position 350 mm with width (Wd) and the height (Hd) of damage are 1 mm and 3 mm respectively. Meanwhile, the FSC is computed for every position and severity, in order to illustrate its variations.

In Figure 4, the FSC is shown as a function of tested position and severity where color levels represent the FSC values. The minimum value (coordinates and value of the minimum) allows the identification of given

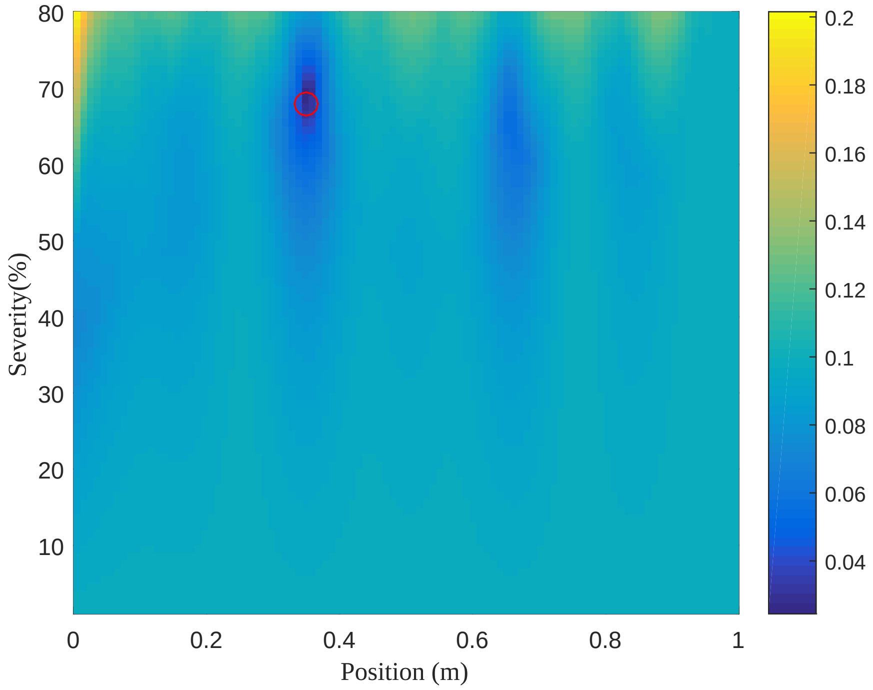


**Figure 4 –** FSC as a function of tested position and severity. The red circle indicates the severity and position identified by the algorithm.

damage in 2D clamped free beam. In this case, a defect of 68% severity localized at 350 mm is found : the position is thus well identified by the FSC and the identified severity corresponds to the parameters chosen for the rectangular damage. These values relate to the other damages properties and information about the size and type. In addition, Table 2 shows the identified damage properties for this particular case of rectangular damage. This is one item of the damaga library the presented strategy is intended to build.

| **Type** | **Rectangular** |
|---|---|
| Position | 350 mm |
| Width | 24.9 mm |
| Length | 1 mm |
| Height | 3 mm |
| Severity | 68 % |

**Table 2 –** Geometry damage properties

## 5 Conclusion

This paper presents a method to identify damage in structure by using natural frequencies. The formulation of the method based on stiffness reduction has been validated with the localization and quantification of the rectangular geometry damage in beam like structure. The simulation correlation with COMSOL and MATLAB are presented and the robustness of the present method is examined. A numerical example with 3D geometry damage case is identified. Based on natural frequency, the damage localization and quantification is accurate because of the sensitivity of the frequency shifts to the damage states. Both 2D and 3D models of the beam were used to link the size of damage to the reduction in stiffness. Geometrical damage properties were successfully accomplished by linking FE models. In the future, more experiments and simulations should be investigated in order to validate the methodology in real cases.

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