Coupled bending torsional vibrations of non-ideal energy source rotors going through critical speeds

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Abstract

With the increasing number of rotors working at very high angular velocity, it becomes crucial to understand the dynamic behavior of rotating machineries when going through critical speeds. Models assuming constant velocity speed are not valid in this case of study since crossing the critical speeds implies necessarily non-stationary working conditions. The present work offers a new finite element model for rotors working at non-stationary regime. The rotational speed is introduced to the unknowns of the dynamic problem and six degrees of freedom are considered on each node. A main focus is given to the study of the coupling between the torsional and flexural degrees of freedom. This coupling is introduced by the intrinsic gyroscopic effect as well as the mass unbalance terms. It results in torsional vibrations containing frequency components of twice the excitation frequency of the mass unbalance as well as frequency components reflecting a modulation with the first bending natural frequency. We show that when crossing the critical speed, an additional frequency component of four times the lateral excitaion frequency appears. The coupling is observed through the analytical equations of motion and confirmed by the numerical simulation.

1 Introduction:

The majority of studies carried on rotordynamics focus either in the lateral behavior of rotating machineries or in the torsional behavior in seperate ways [1, 2, 3]. Fewer studies has been performed for the exploration of coupled bending-torsional behavior. However, those studies for coupled behavior are usually made under some simplifying assupmtions. The mutual influence between transverse and torsional behavior may occur due to several reasons. The gyroscopic effect is the intrinsic source of coupling for rotating machineries as well as the mass unbalance. The coupling between lateral and torsional vibrations in rotors may also arise due to rotor cracks. Another important source of flexural-lateral coupling in rotors is the presence of a gears. One of the early studies on this topic was made by Bernasconi. If the coupling means a mutual influence between the lateral and torsional behavior, Bernasconi, in his paper [4], explored only the torsional vibretions induced by transverse ones. Rao et al [5] explored the effect of the presence of gears on the bending vibrations on the case of permanent regime. XYShen et al. [6] studied the coupled behavior of flexible rotor with six degrees of freedom on each node but also limited the study to the stationnary regime for a given speed of rotation Ω. Al-Bedour [7] Studied the particular case of Jeffcot rotor with no gyroscopic effects and explored the coupling induced by the mass unbalance. R.Sukkar [8] studied an unbalanced Jeffcott rotor but this time in the presence of axial load at stationnary and non-stationnary operating conditions. The aim of this paper is to present a innovative fully coupled model for the study of non-ideal energy source rotors at non-stationnary regime. The speed of rotation of the rotor is considered as an unknown of the dynamic problem and is included in such way that it combines at the same time the nominal rigid body rotation Ω and the torsional deformation θ_t as following:

$$\theta_z = \Omega + \theta_t \tag{1}$$

This way of introducing the degree of freedom θ_z gives more freedom for the simulation of the rotor behavior under non-stationnary regime and offers a more realistic way for observing the phenomena related to the non-ideal energy source rotors, mainly, the sommerfeld effect [9]. The latter is a phenomena that reflects energy exchanges between the rotational direction and the lateral one and can be observed only if the speed of rotation is included to the unknowns of the dynamic problem.
In this paper, the lateral-torsional coupling is observed through the analitycal equations as well as the numerical results.

2 New model for rotordynamics

In this section, we consider a basic rotor made of a shaft, a disk and linear bearings. The excitation of the rotor is due to the mass unbalance.

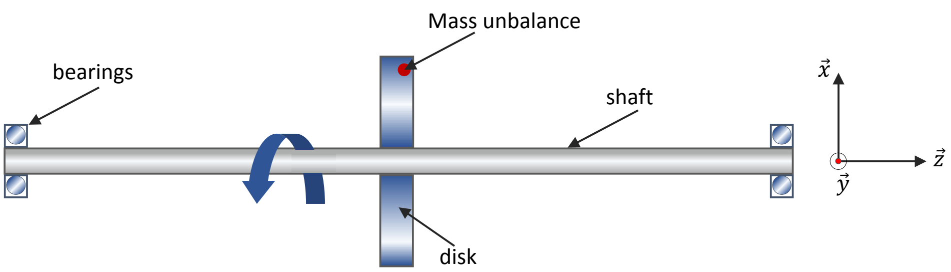

Figure 1: Illustration of the rotor

The finite element method is used to write the equation of motion over a shaft element under the considerd assumptions. Six degrees of freedom are considered on each node. The displacement vector includes the three translations and the three rotation leading to an elementary displacement vector of the following form:

$$\{\delta^e\} = (u_1;\, v_1;\, w_1;\, \theta_{x_1};\, \theta_{y_1};\, \theta_{z_1};u_1;\, v_2;\, w_2;\, \theta_{x_2};\, \theta_{y_2};\, \theta_{z_2})_{\{1,12\}} \tag{2}$$

The energitic approach is adopted to calculate the kinetic and strain energy of the different rotor components. The overall equations of motion are obtained using the Lagrange dynamics and are put in the following form:

$$\begin{aligned}\left(\sum_e [M_s^e] + [M_D] + [M_u]\right)\left\{\ddot{\delta}\right\} + \left(\sum_e [C_s^e] + [G_D]\right)\left\{\dot{\delta}\right\} + \left(\sum_e [K_s^e] + [K_D]\right)\{\delta\} = \\ \{F_{ext}\} + \sum_e [S_s^e] + \sum_e \{F_{se}^{coup}\} + \{F_D^{coup}\} + \{F_{Nlu}\}\end{aligned} \tag{3}$$

$$[S_s^e] = -\ddot{\theta}_{z2}\,[A_1]\,\{\delta^e\} - \dot{\theta}_{z2}\,[Gyr]\left\{\dot{\delta}^e\right\} + \left(\ddot{\theta}_{z2} - \ddot{\theta}_{z1}\right)[A_2]\,\{\delta^e\} + \left(\dot{\theta}_{z2} - \dot{\theta}_{z1}\right)[Gyr^*]\left\{\dot{\delta}^e\right\} \tag{4}$$

$$\begin{aligned}\{F_{s_e}^{coup}\} = -\{N_3^g(l)\}\left(\{\ddot{\delta}^e\}^t\,[A_1^g]\,\{\delta^e\}\right) - \{N_3^g(l)\}\left(\{\dot{\delta}^e\}^t\,[A_1^g]\,\{\dot{\delta}^e\}\right) \\ + \left\{\frac{\partial N_3^g}{dz}\right\}\left(\{\ddot{\delta}^e\}^t\,[A_2^g]\,\{\delta^e\}\right) + \left\{\frac{\partial N_3^g}{dz}\right\}\left(\{\dot{\delta}^e\}^t\,[A_2^g]\,\{\dot{\delta}^e\}\right)\end{aligned} \tag{5}$$

$$[Gyr] = [A_1^g] - [A_1^g]^t \quad ; \quad [Gyr^*] = [A_2^g] - [A_2^g]^t \tag{6}$$

$$[A_1^g] = -2\frac{\rho I_p}{l}\int_0^l \left\{\frac{\partial N_2^g}{\partial z}\right\}\left\{\frac{\partial N_1^g}{\partial z}\right\}^t \mathrm{dz} \quad ; \quad [A_2^g] = -2\frac{\rho I_p}{l}\int_0^l \int_0^z \left\{\frac{\partial N_2^g}{\partial z}\right\}\left\{\frac{\partial N_1^g}{\partial z}\right\}^t \mathrm{dz} \tag{7}$$

$$\begin{cases} u(z,t) = \{N_1(z)\}^t\{\delta_u^e(t)\} & ; \quad w(z,t) = \{N_3(z)\}^t\{\delta_w^e(t)\} \\ v(z,t) = \{N_2(z)\}^t\{\delta_v^e(t)\} & ; \quad \theta_z(z,t) = \{N_3(z)\}^t\{\delta_{\theta_z}^e(t)\} \end{cases} \quad (8)$$

Where $[M_s^e]$, $[C_s^e]$, $[K_s^e]$ and $[Gyr]$ are the classical mass, stiffness, damping and Gyroscopic effect matrix. Matrix $[A_1]$ is the 'stiffness matrix' resulting from the assumption of the non-stationnary regime.$\{F_{ext}\}$ is the vector of external efforts and $\{p^e\}$ is the effort applied by the neighbouring elements on the considered one. Matrices $[A_2]$ and $[Gyr^*]$ are related to the gyroscopic effect under non-stationnary regime and would vanish if the torsional deformation is neglected in the study. Finally, the vector $\{F_{se}^{coup}\}$ derives also from the gyroscopic effect. More details about the vectors and matrices used in the previous equations is presented in [10] where it was explained that the gyroscpic effect terms are taking this form due to the considered assumptions of non-ideal energy source, non-stationnary regime as well as the introduction of torsion in the study. The analytical formulation shows coupling between the lateral and torsional displacements induced by both the mass unbalance and gyroscopic effect. This coupling will be later explored through numerical simulations.

2.1 Numerical results

We consider the following rotor made of a shaft, a disk, linear bearings and elastic coupling (see fig.2). The rotor is excited with a mass unbalance situated on the disk.

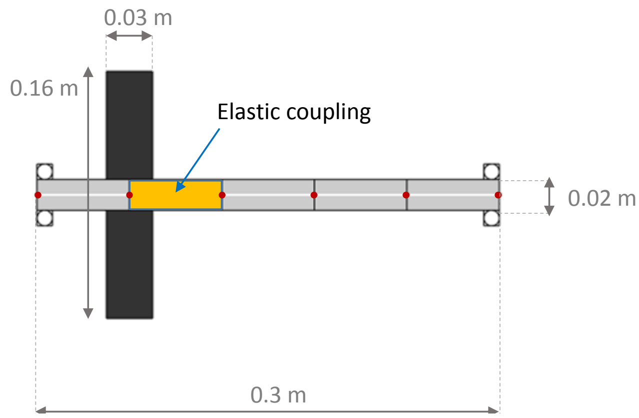

Figure 2: studied rotor

The campbell diagram of the rotor shows two crtical speeds in the speed range between $0rpm$ and $45000rpm$. The first critical speed $\Omega_{cr_1} = 2038\,rpm$ and the second one is at $\Omega_{cr_2} = 20388\,rpm$ (see fig.3).

A linear torque is induced to the rotor. As we can see on the results for the evolution of the instantanious angular speed as a function of time (see fig.4), the sommerfeld effect takes place when the rotor crosses the second critical speed. In this case, the energy induced to the rotor is no longer used to increase its speed but is communicated to the transverse vibrations[11, 12, 13]. The sommerfeld phenomena is usually indesirable because it causes high lateral vibrations magnitude if not enough damping is present in the structure. It is a manifestation of the ineraction between the lateral and rotational direction resulting from the lateral-torsional coupling.The sommerfeld effect couldn't be observed if the assumption of non-ideal energy source wouldn't have been made for the modeling. If not taken into consideration, simulations may lead to an under-estimation of the lateral vibration.

To see more clearly the coupling between the flexural and torsional displacements, we perfom time-frequency anylsis to the torsional deformation on the soft element as shown in figure 5. The torsional vibration signal contains frequencies corresponding to the frequency of excitation relative to the mass unbalance f_u as well as the bisynchronous frequency $2f_u$. Also frequencies of $f_u - f_{b_1}$ and $f_u + f_{b_1}$ are contained in the torsional displacement signal, such as f_{b_1} is the transverse natural frequency at Ω_{cr_2} wich is given by $f_{b_1} = 154\,Hz$. Finally, only when crossing the second critical speed, a frequency of $4 f_u$ is observed in the time-frequency analysis. This highlights the possibility of the transverse vibrations to induce torsional ones. The behavior of the torsional vibrations induced by the lateral ones is slightly different when crossing a critical speed in the presence of the sommerfeld effect as an extra frequency component of $4 f_u$ contibutes to the composition of the torsional signal.

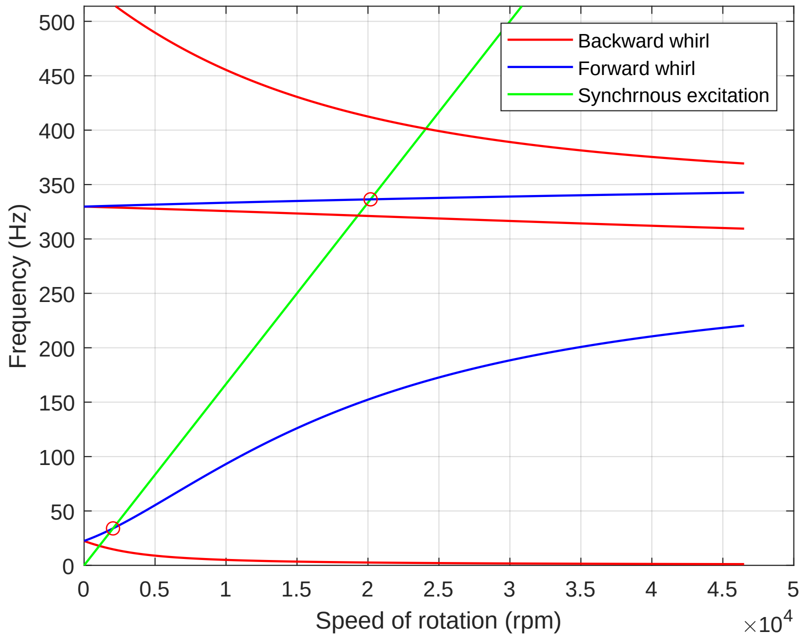

Figure 3: Campbell diagram

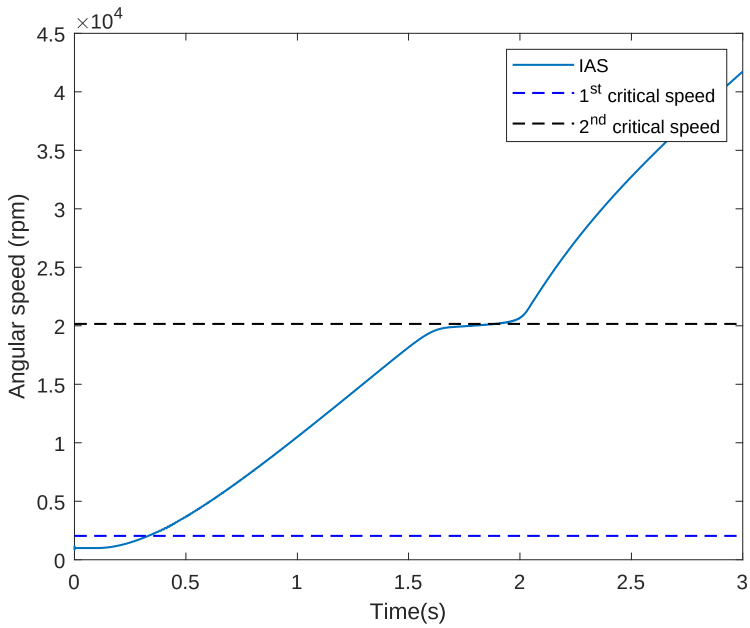

Figure 4: Angular velocity as a function of time

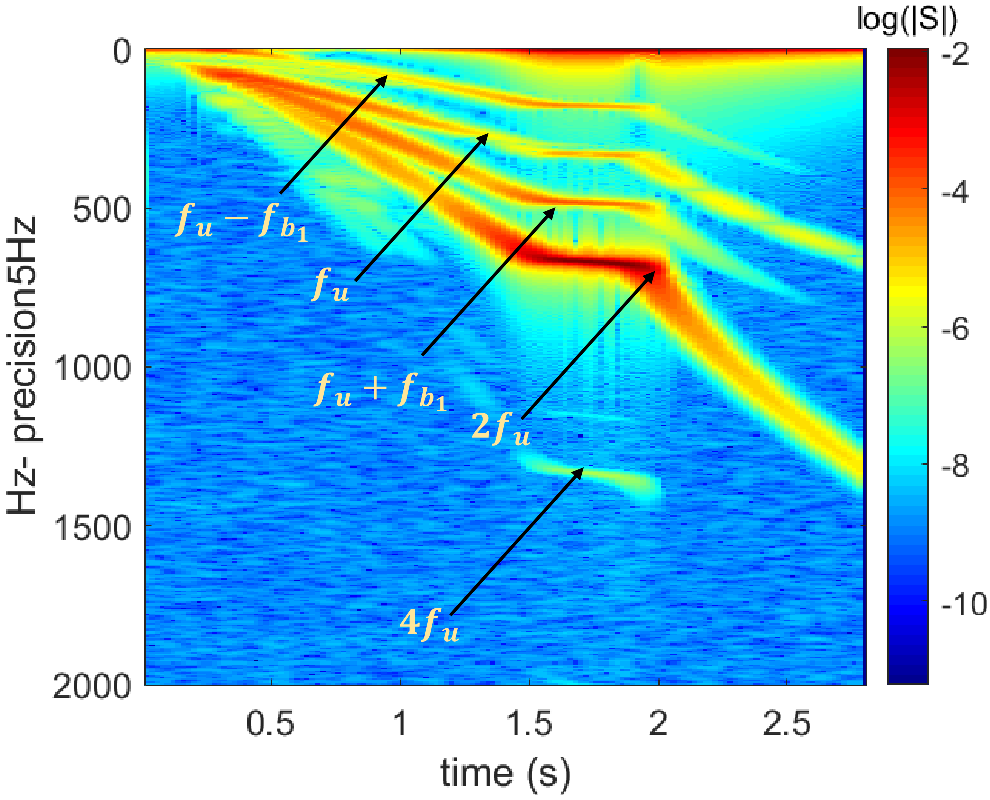

Figure 5: Time-frequency analysis of torsional vibrations on the second element

3 Conclusion

A new finite element model with six degrees of freedom per node is developed for the dynamic response analysis of a rotor systems operating at non-stationnary regime. The proposed new dynamic model is build under the assumption of non-ideal energy source and is a fully coupled lateral-torsional model. The coupling is introduced by both the intrinsic gyroscopic effect as well as the mass unbalance. The time-frequency analysis show that the torsional behaviour is slightly different when crossing a critical speed in the presence of the sommerfeld effect.

The model is build under the less constraining assumptios which extends its use to large case studies. It can be easily extended to the study of multiple rotors connected between each others by elastic coupling.

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