# Influence of Gaussian Signal Distribution Error on Random Vibration Fatigue Calculation.

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## Abstract

In the study of random vibration problems, Gaussian vibration and non-Gaussian vibrations are usually classified according to the excitation signal. The skewness and kurtosis are usually used to distinguish. Here we discuss a non-strict Gaussian signal, which is the error that exists in skewness and kurtosis and usually unavoidable in actual experiments or signals analysis. Through experiments and simulation calculations, the influence of this error on the traditional fatigue calculation method is discussed. The PSD approach will be discussed primarily, and time domain signals based on the rain-flow counting method will be recorded and verified. Total nine calculation model studied in this process. Finally, through a threshold, the range of skewness and kurtosis is indicated, that within this range, Gaussian signal-based calculations can be continued. By comparing the performance of different methods, a better method for signal adaptability can be obtained.

Keywords: Random vibration fatigue, Damage cumulative calculation and Gaussian random vibration.

# 1. Introduction

For many mechanical components, the working load is in the form of random vibrations. In the fatigue design project, the load cycle of the structure is usually obtained by the rain flow counting method according to the conventional time domain signal, with the material property, the damage of the structure could be obtained by using the Miner's Law, and then prediction the life of the structure. But the acquisition of time domain signals relies on a large number of experimental records, which obviously sounds expensive. Later, according to the stochastic theoretical method, the power spectral density was used to characterize the random vibration characteristics, and the method of inferring the rain flow counting result of response stress was proposed.[1][2] This theory generally assumes that the load is subject to a Gaussian distribution. The more successful method is the narrow-band approximation method proposed by Bendat in 1968.[3] Later, due to the efforts of more scholars, the broadband approximation method was also proposed. At the same time, various improvement schemes were proposed to improve the accuracy of the approximate results. [4][5][6]

However, since most of the actual loads are non-Gaussian distributions, it is obvious. Therefore, when the load is a non-Gaussian signal, the original Gaussian-based frequency domain damage analysis method may be used, which may cause poor deviation. Therefore, it is necessary to further discuss the influence of signal non-Gaussian on stress distribution. The research method based on the frequency domain signal discusses the overall distribution of signals in the frequency domain, and the non-Gaussian signal that satisfies this condition is not unique. This leads to the use of frequency domain method to study the distribution of non-Gaussian signals. Large deviations, when calculating damage using this distribution result, pose a significant risk to product design and life estimation.

In this paper, a reference based on kurtosis judgment will be proposed to select the method of fatigue damage calculation. The rain-flow count analysis is performed on the response stress of non-Gaussian load, and the difference between the Gaussian signal and the non-Gaussian signal at the same level is obtained, and an allowable value is obtained, that is, in this range, even if it is not a Gaussian signal, the PSD method also could be used. The fatigue damage results obtained by the method are still within error tolerance. Beyond this range, the non-Gaussian signals must be considered with the special method.

# 2. Non-Gaussian signal and Kurtosis control

Generally, a signal whose probability density distribution obeys a Gaussian distribution is called a Gaussian signal and is mainly judged by the skewness and kurtosis of the signal. This indicator indicates the distribution of data within the data range. The skewness refers to the zero offsets of the centre, which is represented by S. The kurtosis can be understood as the specific gravity in the central region, denoted by K. The greater the kurtosis, the greater the accumulation of data in the centre. Usually, the Gaussian distribution has a kurtosis of 3 and skewness of 0.

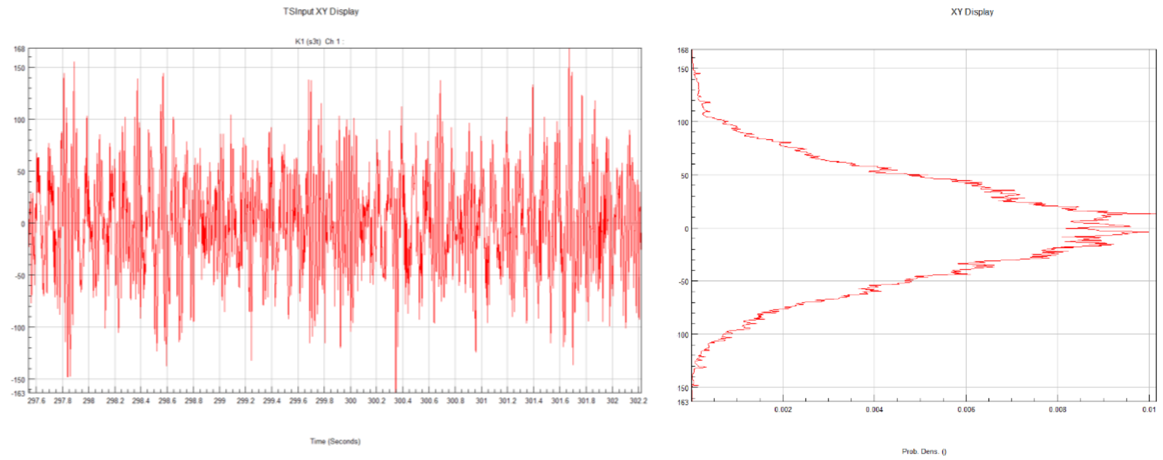


Figure 1 Gaussian signal and probability density distribution

Thus $\sigma$ is the standard deviation, $\mu_n$ is the nth central moment

$$S = \frac{\mu_3}{\sigma^3} \tag{1}$$

$$K = \frac{\mu_4}{\sigma^4} \tag{2}$$

When the kurtosis or skewness has a condition that does not satisfy the Gaussian distribution, the signal is called a non-Gaussian distribution. At the same time, according to the index, when K>3, it is called leptokurtic, and when k<3, it is called platykurtic. According to the probability density distribution of the non-Gaussian signal, it can be found that the kurtosis reflects the distribution of acceleration in the middle region.[7]

In order to obtain an acceleration signal of a non-Gaussian distribution, it is usually obtained by Gaussian signal transformation. There are many methods used, Hermit polynomial, Gaussian mixture model, Phase selection method, Power-law model, Exponential method, Non-parametric method. In this paper, Steinwolf's phase selection method is used to modulate non-Gaussian signals with specific skewness and kurtosis.[8]

# 3. Phase selection method

By fast Fourier transform, an acceleration time domain signal can be described as a superposition of harmonics in the frequency range.

$$x(t) = \sum_{n=1}^{N} A_n \cos(2\pi n\Delta f t + \varphi_n) \tag{3}$$

The amplitudes of the harmonics are obtained

$$A_n = \sqrt{2\Delta f S(n\Delta f)} \tag{4}$$

Then the nth centre moment could be obtained

$$M_z = \lim_{T\to\infty} \frac{1}{T}\int_0^T \{x(t)\}^z dt = \frac{1}{T}\int_0^T \{x(t)\}^z dt, \ \ z > 2 \tag{5}$$

After the kurtosis formula could be written as

$$K = \frac{M_4}{M_2^2} = 3 - \frac{3}{2}\left(\frac{\sum_{n=1}^{N} A_n^4}{\left(\sum_{n=1}^{N} A_n^2\right)^2}\right) + \left(\frac{1}{2}\sum_{n=1}^{N} A_n^2\right)^{-2} \{\frac{3}{2}\sum_{\substack{i+2j=k \\ i\neq j}} A_i {A_j}^2 A_k \cos\left(\varphi_i + 2\varphi_j - \varphi_k\right) + \frac{3}{2}\sum_{\substack{i+j=2k \\ i<j}} A_i {A_k}^2 A_j \cos\left(\varphi_i + \varphi_j - 2\varphi_k\right) + 3\sum_{\substack{i+j=k+m \\ i<j,k<m,i<k}} A_i A_j A_k A_m \cos\left(\varphi_i + \varphi_j - \varphi_k - \varphi_m\right) + 3\sum_{\substack{i+j+k=m \\ i<j<k<m}} A_i A_j A_k A_m \cos\left(\varphi_i + \varphi_j + \varphi_k - \varphi_m\right) + \frac{1}{2}\sum_{i=3j} A_i {A_j}^3 \cos\left(\varphi_i - 3\varphi_j\right)\} \tag{6}$$

In the case of ensuring that the mean and RMS of the non-Gaussian signal are not changed, only the distribution is changed. So to make the K fitted for the experiment by modulating the specific $\varphi$. The resulting non-Gaussian distribution acceleration time series with specific K could be found. It should be noted that as the bandwidth increases，the amount of calculation becomes very large and also affecting the amount is the sampling frequency and data length. In general, the required non-Gaussian data is obtained in a combination of several methods.

## 4. Rain-flow count and damage calculation

The rain-flow counting method was proposed in the 1950s by two British engineers, M. Matsuishi and T. Endo.[9] The main function of this counting method is to simplify the measured load history into several load cycles for fatigue life estimation and fatigue test load spectrum. It is based on the two-parameter method and considers two variables of dynamic strength (magnitude) and static strength (mean). The rain flow counting method is mainly used in the engineering field, and is widely used in the calculation of fatigue life.

Through the rain flow counting method, the response stress is cyclically counted, and the stress amplitude-cycle number curve is obtained, which is the ***p(s)*** curve required for the fatigue damage calculation. The ultimate goal of the frequency domain based PSD method is also to approximate the fitting through various frequency models through the frequency domain information, and finally obtain the probability density curve ***p(s)*** of the stress response.

The S-N curve of the material expresses the number of life cycles of the material under different stresses. Usually expressed in the Basquin model.[10]

$$S^m N = C \tag{7}$$

And then according Miner's Law[11], calculation formula of damage in unit time could be written

$$D = v_p C^{-1} t_0 \int_0^\infty S^m p(s) ds \tag{8}$$

Then the prediction life could be obtained by D

$$T = \frac{1}{D} \times t_0 \tag{9}$$

## 5. Case study

In order to study the influence of the probability distribution of the response stress on the structural damage calculation, using the same frequency range, the 0 mean, the RMS is the same, the skewness is 0, and the different time series with the kurtosis between 2.9-10 are applied to the same double notched specimen, the effect of kurtosis on the damage calculation is obtained by comparing the damage conditions.

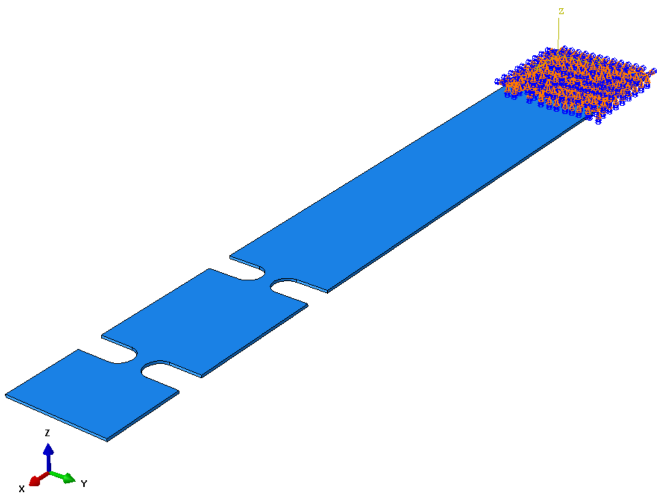


Figure 2 Two notch specimen & excitation

To meet the requirement, 10s time series of acceleration which allow non-Gaussian distribution were generated with different kurtosis. The bandwidth selection is based on the modal analysis result of the specimen, the second-order bending mode is 51.2 Hz, and 15-95 Hz is selected as the frequency range of the excitation signal. The sampling ratio is 1024Hz according to the band width.

| Frequency | PSD ($g^2$/Hz) | ($m^2/s^4$)/Hz |
|---|---|---|
| 15 | 0.25 | 12.03 |
| 95 | 0.25 | 12.03 |
| RMS | 4.472136g | 31.02m/$s^2$ |

Table 1 Frequency domain information (g=9.81 m/$s^2$)

The simulation part is finished by Abaqus. Before taken simulation, the modal analysis was used to check FE model to do verify and validation according mode frequency and stress distribution. Though the modal dynamic, the response stress of different times series could be obtained.

| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| RMS(m/$s^2$) | 31.44 | 31.32 | 31.27 | 31.31 | 31.10 | 30.84 | 31.54 | 31.44 |
| Kurtosis | 3.01 | 3.50 | 4.01 | 4.53 | 5.02 | 8.04 | 10.24 | 2.85 |
| No. | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| RMS(m/$s^2$) | 31.44 | 31.44 | 31.44 | 31.44 | 31.44 | 31.44 | 31.40 | 31.52 |
| Kurtosis | 2.92 | 2.80 | 3.06 | 2.96 | 3.12 | 2.90 | 3.15 | 3.21 |

Table 2 The RMS and Kurtosis of acceleration

After, the response stress result was show that the specimen is not linear structure, the time series of response stress was as below. The direction opposite to the gravitational acceleration is the Z-axis forward direction, and the nodal stress at the left side of the upper surface of the model is selected as the research object.

The overall research strategy is shown below. The focus is on the difference in stress cycling and damage values after rain flow counting. In general, the infinite fatigue stress of the material is selected as the threshold value of the rain flow count, that is, in this stress cycle and below, the material has an infinite life. However, in order to compare the results of the data, this threshold is set to zero.

## 6. Results

According the result of response stress, it could be found that the kurtosis were influence by structure. Moreover, as the acceleration kurtosis increases, the kurtosis of the response stress decays more significantly.

| No. | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|---|
| RMS(MPa) | 28.89 | 29.00 | 28.88 | 29.74 | 29.39 | 30.21 | 31.77 | 29.07 |
| Kurtosis | 3.07 | 3.30 | 3.52 | 3.92 | 3.70 | 6.11 | 6.49 | 2.85 |
| No. | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 |
| RMS(MPa) | 28.89 | 29.09 | 29.17 | 28.34 | 28.89 | 28.45 | 29.00 | 28.93 |
| Kurtosis | 3.02 | 2.78 | 2.90 | 3.07 | 3.88 | 3.14 | 3.04 | 3.18 |

Table 3 The RMS and Kurtosis of response stress

By comparing the results of time-domain and frequency-domain methods with different kurtosis, it can be found that as the kurtosis increases, the stress distribution is more concentrated in the low-stress region, but the maximum value of the stress amplitude is significantly increased. The key to damage deviation. Moreover, there is a possibility that the maximum value of the stress amplitude is greater than the allowable stress directly causing structural failure.

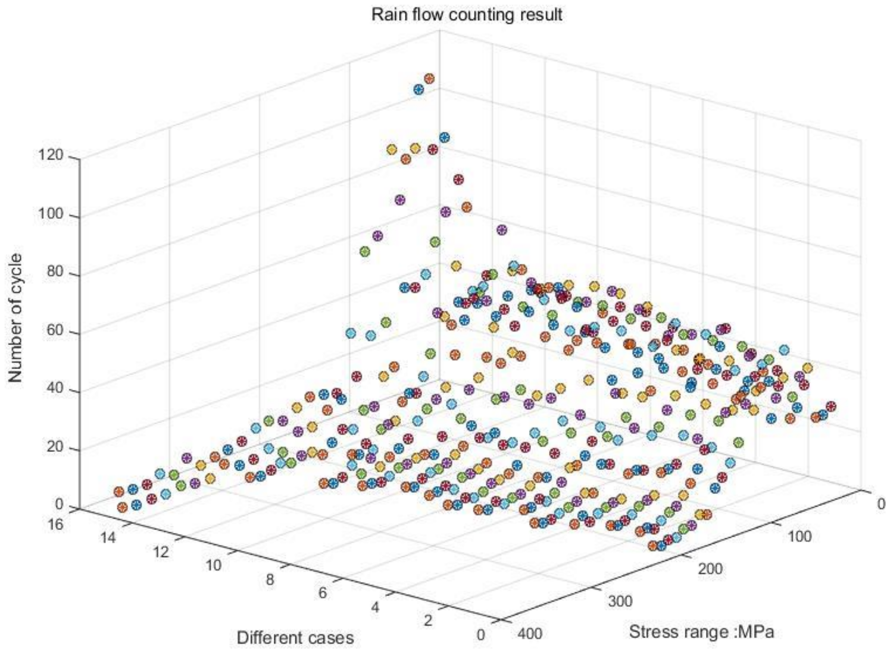


Figure 3 Rain flow counting result

The DIRLIK method is selected to obtain the stress expectation based on the Gaussian distribution of the frequency domain method, which is used as a reference object and compared with the results under the time domain signal.

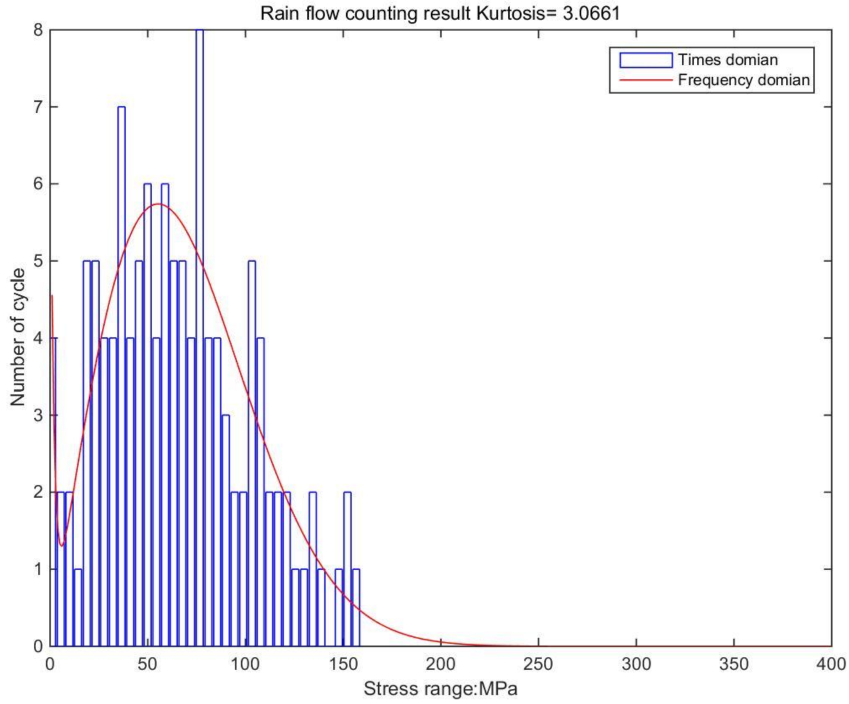


Figure 4 Rain flow counting of K=3.07

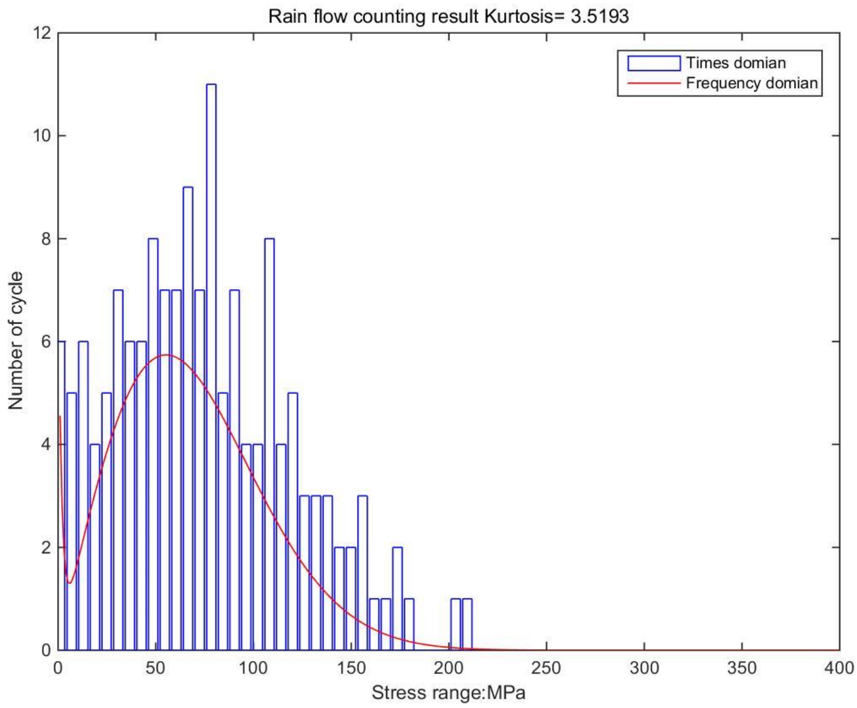


Figure 5 Rain flow counting of K=3.52

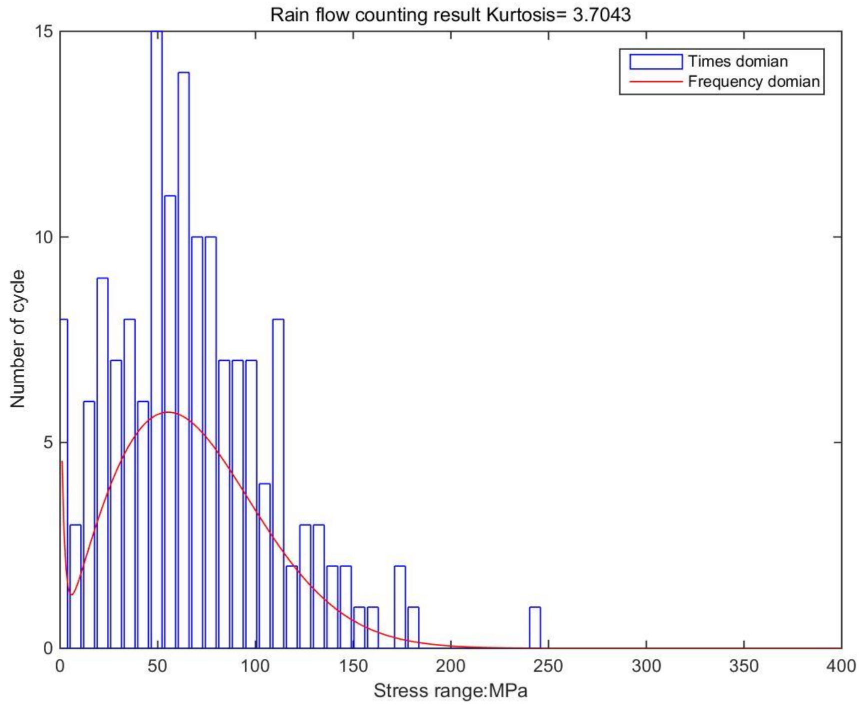


Figure 6 Rain flow counting of K=3.70

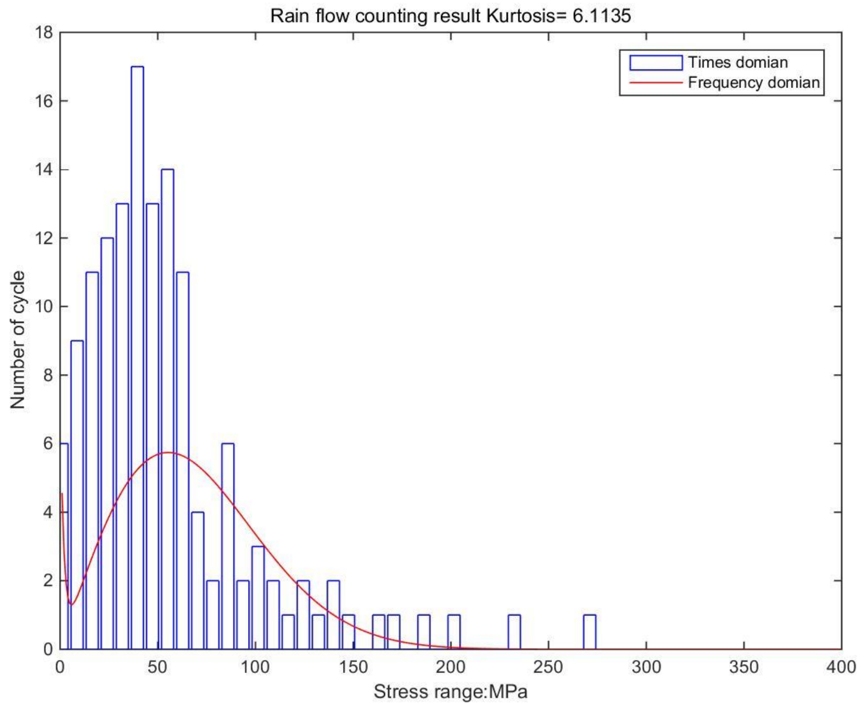


Figure 7 Rain flow counting of K=6.11

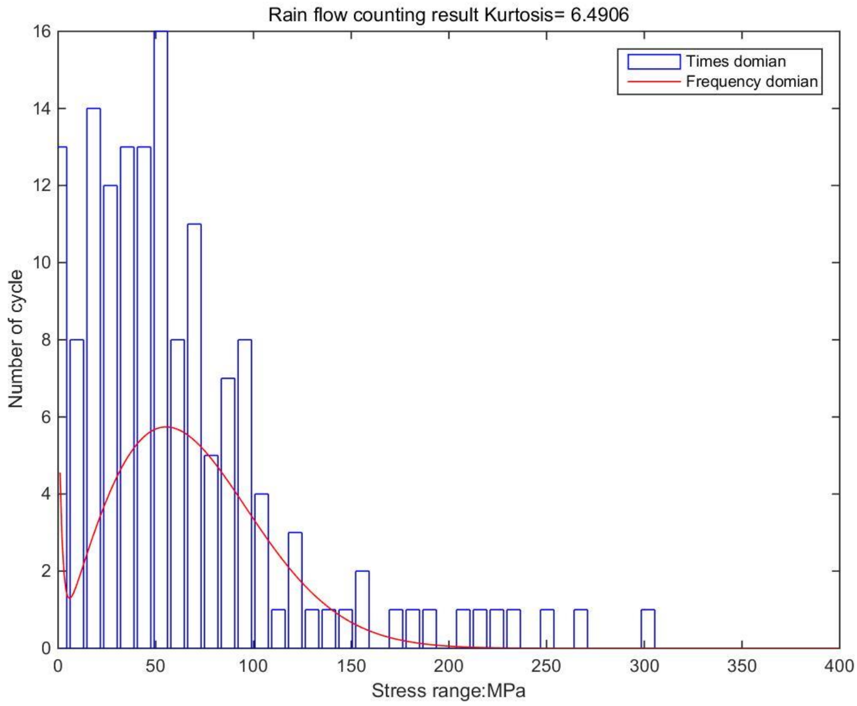


Figure 8 Rain flow counting of K=6.49

Though Miner's Law, the linear cumulative damager could be calculation,

$$D = \frac{n}{N} \tag{10}$$

For the different kurtosis time series data, the error control $\gamma$ could be written ,

$$\gamma = 1 - \frac{D_K}{D_{PSD}} = \frac{\sum_0^{S_{max}} \cdot N_k(s) \cdot s^m}{v_p \int_0^{\infty} s^m p(s) ds} \tag{11}$$

When $\gamma \leq 30\%$, this can accept the results of using a PSD-based Gaussian signal for non-strict Gaussian vibration signals. Beyond this range, fatigue calculation methods based on non-Gaussian vibration signals must be used. Generally, when the kurtosis is less than 3.5, the result is within an acceptable range. Of course the accuracy of this result is limited by the length of the data. The longer the data, the more obvious the distribution. Smaller samples are relatively more affected by randomness.

# 7. Conclusion

Based on the analysis results, it can be found that the data difference is affected by the material property $\boldsymbol{m}$. At the same time, due to the structural relationship, the actual excitation kurtosis is greater than the threshold, but the response stress after attenuation can be calculated according to the Gaussian distribution.

Because of the high kurtosis, the response stress exceeds the limit of material strength, and should be avoided in industrial design. For response stresses with high kurtosis, there is a possibility that the maximum value exceeds the strength limit. This should be taken seriously.

# 8. Acknowledgement

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