Characterization of the damping added by a foam on a plate by an inverse vibration problem

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Abstract
The industrial solution used today to improve the acoustic performance of a structure is often ensured by the addition of damping treatments such as elastomer, PVC or bituminous. In the transportation, these materials increase the costs and mass of the vehicle and have a negative environmental impact. Unlike theses heavy materials, it is commonly known in the professional standards of automotive designers that foams in vehicle trim provide damping to the structure. It has been shown that some impregnated PU foam coatings provide significant and equivalent damping to conventional bituminous materials used in the automotive industry. This observation makes it possible to extend the function of the trim to vibration damping, in order to mutualize the both problem (acoustic and vibration) in one treatment. To understand and quantify this dissipation mechanism involved by adding a porous material to a supporting structure [1], it is proposed to treat the problem from the angle of an experimental quantification obtained by an inverse problem. The proposed approach is based on the use of the Force Analysis Technique (FAT) method [2] [3], where the first objective is to locate and quantify the forces applied to a vibrating structure. In this case, the FAT is designed to define the damping provided by the foam to the vehicle trim [4] [5].
Keywords : foam, damping, FAT method

1 Introduction

The transport industry is constrained by vehicle weight reduction and the resulting impact on performance. One of the solutions available is the additional damping treatments such as elastomer, PVC or bituminous. But, they increase the weight of the vehicle and reduce the benefits of lightening solutions (use of thermoplastic shell instead of steel panel, reduction in steel thicknesses used for the structure). In previous studies, some impregnated PU foam coatings provide significant damping to the structure. The viscoelastic damping of these materials in tension-compression is low and does not explain these performances. The origin of this damping can be generated by the dry friction at the interface between the foam and the structure. The objective of the study is also to dissociate the part of the damping generated by the viscoelasticity of the foam and generated by the relative displacement of the material with respect to the supporting structure.

To measure the dry friction damping at the interface, the Force Analysis Technique (FAT) method [6] [4] is used to solve the equation of plate motion. With this local method, it is possible to find the damping of the foam-plate structure in the medium and high frequencies. The Corrected Force Analysis Technique (CFAT) method [7] [5] is used in high frequencies to improve measurements. A combination of these two methods increases the frequency range studied. The system studied is a viscoelastic material (PU foam) placed on a steel plate. Using a LASER vibrometer, the foam-plate contact area is swept to measure the vibration field. The results obtained for the loss factor and the stiffness term are given for with and without foam on the plate.

2 Theory

2.1 Equation of motion of a plate

The equation of motion for a thin isotropic plate in harmonic regime for bending stress is

$$D\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial y^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}\right)-\rho h\omega^2 w=F(x,y,\omega), \tag{1}$$

where D is the bending stiffness, ρ the density, h the thickness, ω the angular frequency, $w(x,y)$ the transverse displacement field and $F(x,y,\omega)$ the distribution of the external forces exerted on the plate. Flexural rigidity

$$D=\frac{E(1+j\eta)h^3}{12(1-\nu^2)}, \tag{2}$$

introduces the Poisson coefficient *nu* and the Young's modulus complex $E(1+j\eta)$ where η is the loss factor, which is the damping factor of the system. The equation of motion (1) is called local, because it is valid at any point in the structure and independent of boundary conditions. Considering an area where no force is applied ($F(x,y,\omega)=0$), the equation (1) becomes

$$\frac{D}{\rho h\omega^2}\left(\frac{\partial^4 w}{\partial x^4}+\frac{\partial^4 w}{\partial y^4}+2\frac{\partial^4 w}{\partial x^2\partial y^2}\right)=w. \tag{3}$$

Knowing the displacement field $w(x,y)$ of the structure and its spatial derivatives $\frac{\partial^4 w}{\partial x^4}$, $\frac{\partial^4 w}{\partial y^4}$ and $\frac{\partial y^4 w}{\partial x^2\partial y^2}$, it is possible to identify the term $\frac{D}{\rho h\omega^2}$, which may vary with the frequency. The real and imaginary parts of this term give the stiffness and damping of the structure. The characteristics of the structure can be determined by a measurement of the displacement field and the estimation of spatial derivatives, however the measurements made are noisy. The FAT and CFAT methods are used to reduce measurement noise and retrieve essential information.

2.2 CFAT and FAT methods

The CFAT method regularizes the inverse resolution using the natural filter of discretization by finite differences in the equation of motion. The CFAT method initially allows to find the force distribution on a known structure using the measured displacement field, in the case it is not the force distribution, but the characteristics of the structure which are studied. This method consists in introducing correction coefficients into the equation of motion, in order to benefit from the filtering effect of the finite difference scheme while correcting the bias it introduces into the resolution. The equation of discretized motion corrected

$$\frac{D}{\rho h\omega^2}\left(\tilde{\mu}^4\delta_{ij}^{4x}+2\tilde{\nu}^4\delta_{ij}^{2x2y}+\tilde{\mu}^4\delta_{ij}^{4y}\right)=w_{ij} \tag{4}$$

presents schemas with finite differences

$$\begin{aligned}
\delta_{ij}^{4x} &= \frac{1}{\Delta x^4}(w_{i+2,j}-4w_{i+1,j}+6w_{i,j}-4w_{i-1,j}+w_{i-2,j}),\\
\delta_{ij}^{4y} &= \frac{1}{\Delta y^4}(w_{i+2,j}-4w_{i+1,j}+6w_{i,j}-4w_{i-1,j}+w_{i-2,j}),\\
\delta_{ij}^{2x2y} &= \frac{1}{\Delta x^2\Delta y^2}(w_{i+1,j+1}-2w_{i+1,j}+w_{i+1,j}-2w_{i,j+1}+4w_{i,j}-2w_{i,j-1}+w_{i-1,j}+2w_{i-1,j}+w_{i-1,j-1}),
\end{aligned}$$

and corrective coefficients

$$\begin{aligned}
\tilde{\mu}^4 &= \frac{\Delta^4 k_f^4}{4[1-\cos(k_f\Delta)]^2},\\
\tilde{\nu}^4 &= \frac{\Delta^4 k_f^4}{8[1-\cos\left(\frac{k_f\Delta}{\sqrt{2}}\right)]^2}-\tilde{\mu}^4.
\end{aligned}$$

The corrective terms require knowing the number of natural bending waves of the plate

$$k_f^4 = \frac{\rho h}{D}\omega^2. \tag{5}$$

This wave number k_f is unknown because it depends on the characteristics of the plate. Characteristics are identified iteratively, with a first iteration without correction (i.e. by imposing $\tilde{\mu}^4 = \tilde{\nu}^4 = 1$), to provide a first initial value of k_f. At each iteration the value of $\frac{D}{\rho h}$ is identified, which provides a new estimate of k_f. According to the system, the number of iterations is not the same, in our case 10 iterations are necessary to ensure the convergence of the inverse problem. The advantage of the CFAT method is that the regularization is done automatically during the inverse resolution. It is not necessary to calibrate a regulation parameter, unlike the RIFF method. However, it has a certain validity range given by

$$\begin{array}{l|l} f^{min}(\Delta) = \frac{\pi}{8\Delta^2}\sqrt{\frac{Eh^2}{12\rho(1-\nu^2)}} & f^{min}(0.0133) = 3.985\cdot 10^3 \text{ Hz}, \\ f^{max}(\Delta) = \frac{\pi}{2\Delta^2}\sqrt{\frac{Eh^2}{12\rho(1-\nu^2)}} & f^{max}(0.0133) = 1.594\cdot 10^4 \text{ Hz}, \end{array}$$

where Δ represents the spatial discretization between two points. To identify the characteristics of the plate over the entire frequency range, it is necessary to use in combination with the RIFF method for low frequencies.

The FAT method is used to regularize errors due to measurement noise using a low-pass filter in wave number. First of all, it is necessary to window the signal to soften the discontinuities at the limits and avoid the negative effects of the filter (Gibbs phenomenon). This is done using a Tukey window

$$\Psi_{i,j}^{2D} = \Psi^{1D}(x_{i,j} - x_{1,1} - 2\Delta_x)\cdot\Psi^{1D}(y_{i,j} - y_{1,1} - 2\Delta_y), \tag{6}$$

where

$$\Psi^{1D}(x) = \begin{cases} 0.5\left(1-\cos\left(\frac{\pi x}{\alpha}\right)\right) & \text{si } 0 \le x < \alpha, \\ 1 & \text{si } \alpha \le x < L-\alpha, \\ 0.5\left(1-\cos\left(\frac{\pi(x-L+2\alpha)}{\alpha}\right)\right) & \text{si } L-\alpha < x \le L, \\ 0 & \text{sinon}\ . \end{cases} \tag{7}$$

with $L = L_x - 4\Delta_x$ and

$$\alpha = \begin{cases} \lambda_c & \text{si } L \ge 2\lambda_c, \\ \frac{L}{2} & \textit{sinon}. \end{cases}$$

The filter, which removes high wave numbers, is weighted by a Hanning window to keep the local aspect of the method. This filter eliminates the amplification of errors associated with the inverse problem. The spatial response of the filter

$$h(x,y) = \begin{cases} \frac{k_c^2}{4\pi^2 xyN_f}\left(1+\cos\left(\frac{k_c x}{2}\right)\right)\left(1+\cos\left(\frac{k_c y}{2}\right)\right)\sin(k_c x)\sin(k_c y) & \text{si } x \text{ et } y\varepsilon\left[\frac{-2\pi}{k_c}, \frac{2\pi}{k_c}\right], \\ 0 & \text{sinon}\ , \end{cases} \tag{8}$$

introduces a normalization parameter N_f, chosen so that $\iint h(x,y)dxdy = 1$ and a cut-off wave number k_c. The latter is generally chosen in proportion to the natural waves number

$$k_c = a\cdot k_f, \tag{9}$$

where the regulation parameter a is generally set at 4 for very good measurements and 1 for very noisy measurements. In our case, the combination of the FAT method with the CFAT method avoids using a regulation parameter. Here, the cut-off wave number k_c is chosen equal to the bending wave number of the plate k_f determined with the CFAT method.

3 Experimental validation

3.1 Set-up

To characterize the dry friction damping between the foam and the plate, the temperature (19° C) chosen is lower than the glass transition (55° C) of the foam to have the least viscoelastic damping effect. The measurement is made on a structure composed of a steel plate (700x700x2 mm) suspended at the four ends by elastics, to approach conditions at the free limits and a foam plate (40x40 cm) simply placed on the plate. A scanning laser vibrometer is used to measure the displacement field in the study area (56xx56 cm). The vibrometer is located below the steel plate, at a distance of 132 cm . The excitation, of impulse type, is applied using an automatic impactor with an amplitude of 5 N and repeated 3 times for each measurement point. Figure 1 shows a schema of the experimental set-up.

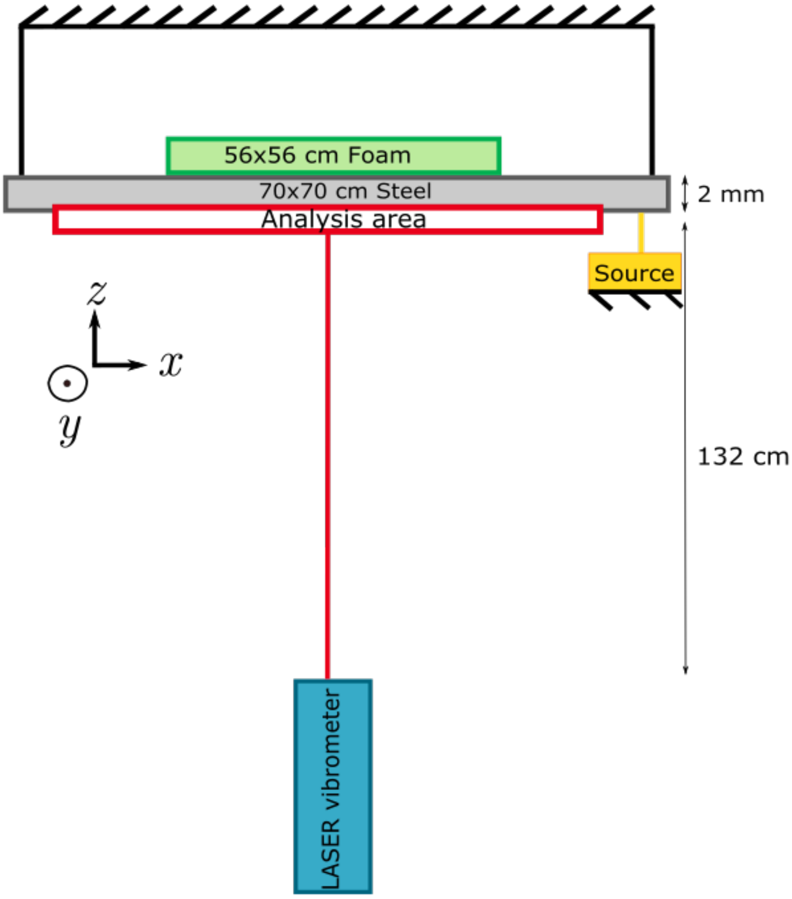

Figure 1: Experimental set-up

The measurement mesh has 33x33 = 1089 points, with a spatial pitch of about 0.13 cm. The studied frequency range is up to 10 kHz and the frequency step is 0.78 Hz. A wide frequency range is observed to measure the effects of foam at low and high frequencies.

The foams used are of the thermoplastic elastomer type, they are normally used for the sound insulation of vehicles. In this case the study extends to the use of these foams for the vibration damping of a steel structure. Different foam thicknesses are tested, but only one is compared with the blade plate. Friction at the interface between the foam and the plate is a dry friction, which may result in a non-linear response of the structure. In this study, it is not the damping at the interface that is identified, but the global damping of the foam-plate structure.

3.2 Results

Two measurements are made to obtain the displacement of the plate without foam, then with 11 mm thick foams. The characteristics obtained for the structure are the stiffness $\Re\left(\frac{D}{\rho h}\right)$ (Figure 2) and shock absorption

$\eta = \frac{\Im\left(\frac{D}{\rho h}\right)}{\Re\left(\frac{D}{\rho h}\right)}$ (Figure 3) for a frequency range from 400 Hz to 10 kHz.

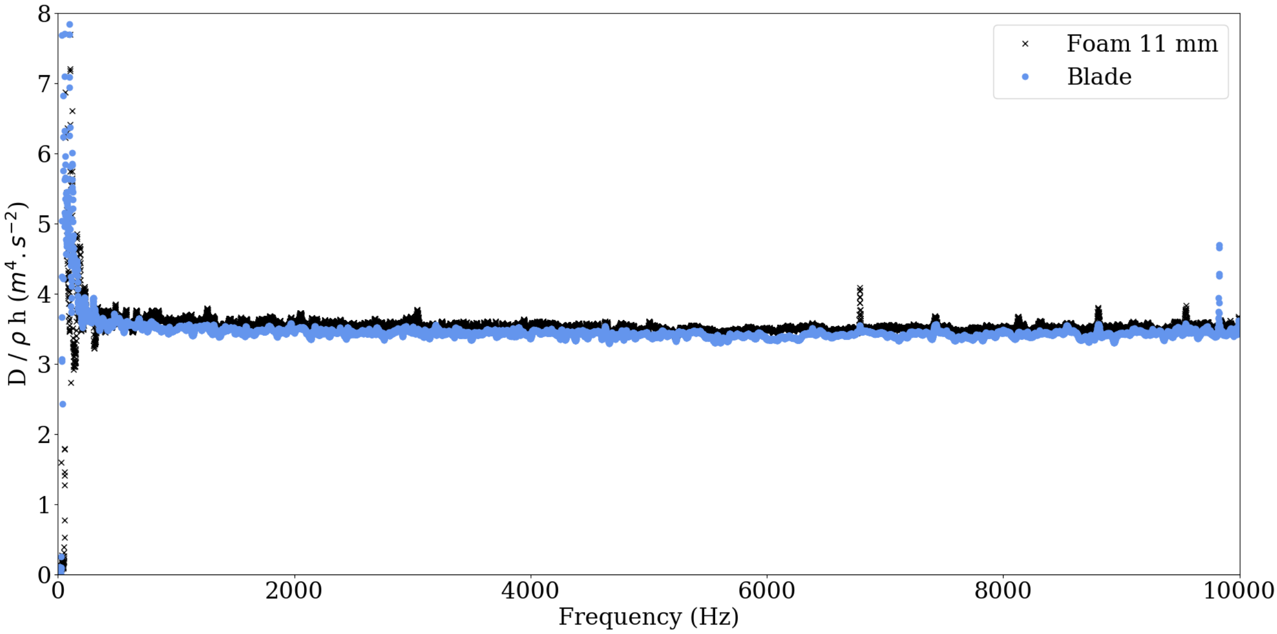

Figure 2: Stiffness $\frac{D}{\rho h}$ of the structure with only the plate and different foam thicknesses

The stiffness for the two configurations varies. The average value of the stiffness is $3,528\ \mathrm{m}^4 \cdot \mathrm{s}^{-2}$ for the configuration with the 11 mm foam and $3,454\ \mathrm{m}^4 \cdot \mathrm{s}^{-2}$ for the bare plate. The difference in stiffness between the configuration with the foam (98 g) of 11 mm and the bare plate is 7%.

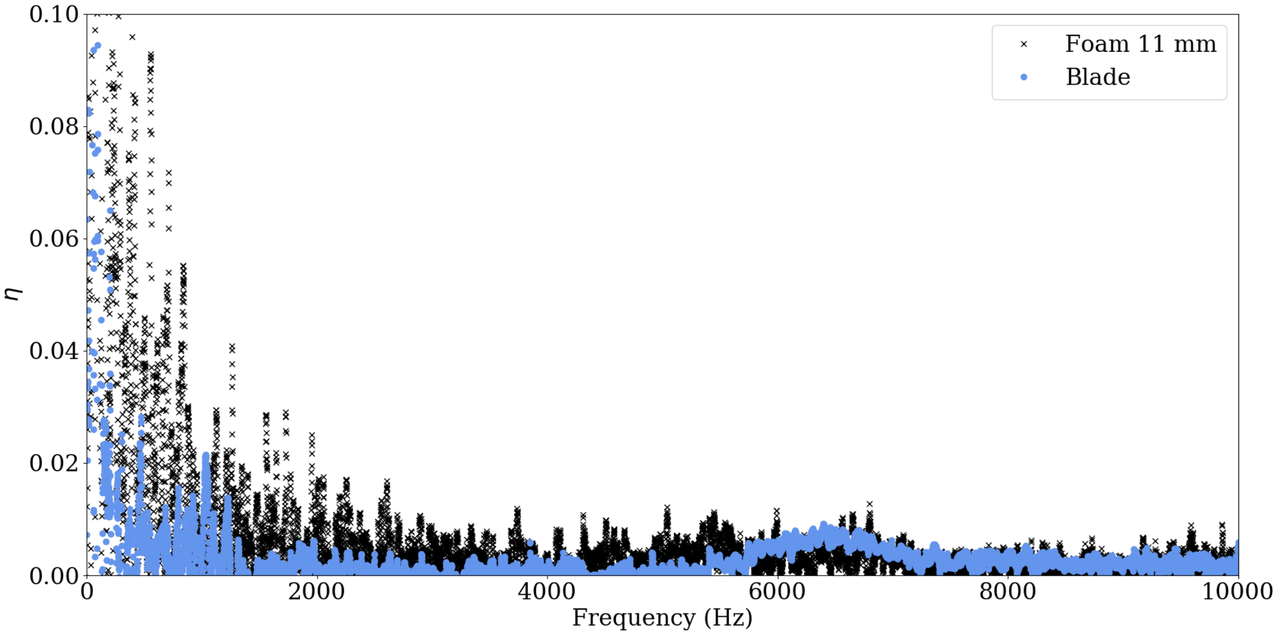

Figure 3: Loss factor η of the structure with only the plate and different foam thicknesses

The damping increases with the foam, mostly at low frequencies. Around 7000 Hz, there is an increase in the damping for the bare plate. This increase may be related to the vibration mode of the hooks used for suspension. Further study would reveal the cause of this increase in absorption.

4 Conclusion

The loss factor obtained after laser vibrometer measurement validates the method (FAT-CFAT) used to determine the damping of foam placed on a steel plate. In addition, the combination of the FAT and CFAT method allows the frequency validity range to be extended compared to only used the CFAT method. So, it isn't necessary to use an adjustment parameter for the FAT-CFAT method. The results obtained make it possible to validate both the experimental procedure and the FAT-CFAT method. In order to determine the origin of the resultant damping, a numerical study is in progress to characterize the damping of the vibration by the viscoelastic effect of the foam. To decide on the origin of the damping, whether viscoelastic or by dry friction, the test is compared to the numerical calculation. In order to increase the measurement dynamics, several modifications of the test method are planned. To limit the damping effect of the free suspension, the use of a system with supported boundary conditions is considered. In order to increase the damping of the system, a decrease in the rigidity of the support plate will be studied.

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