

# Structural Dynamics of Electric Machines Subjected to PWM Excitations

Margaux TOPENOT<sup>1,2</sup>, Morvan OUISSE<sup>1</sup>, Gael CHEVALLIER<sup>1</sup>, Damien VAILLANT<sup>2</sup>

<sup>1</sup>Institut FEMTO-ST, Université de Bourgogne Franche-Comté,  
24 Rue de l'Épitaphe, 25000 Besancon, France

<sup>2</sup>Alstom Transport,  
7 Avenue du Maréchal de Lattre de Tassigny, 25290 Ornans, France

## Abstract

Pulse Width Modulation (PWM) is commonly used for driving asynchronous machines. The mechanical torque is the result of the combination of several multiphysics conversions. The PWM is generally defined on the basis of the electromagnetic performances determined with simple mechanical behavior: typically, rigid bodies are considered to be representative of the mechanical behavior. However, since PWM generates numerous harmonics of current and voltage, it may have an impact on the structural dynamics of the system. In particular, the rotor is subjected to torque oscillations over a wide frequency range that may result in large vibration amplitudes when coincidences occur. These excitations can induce severe damages like fans ruptures. In this work, a finite element model and an reduced model are compared in terms of ability to describe the structural dynamics of the system when the electric machine is driven with PWM. The results are compared and discussed in terms of applicability for the design of electric machines.

## 1 Introduction

Trains are set into motion through electrical energy. Electrical signals are synthesized by the Pulse Width Modulation (PWM). From a theoretical point of view, train motors can be driven by sine wave signals but practically, due to signal synthesis issues and power limitations, motors are driven thanks to voltage impulsions. This allows to manage easily the variation of the rotating speed. The PWM definition makes the fundamental frequency equal to the sine signal frequency. Unfortunately, PWM generates small-amplitude oscillations which frequencies correspond to the harmonics. They lead to vibration motions that engender spurious mechanical stress. Figure 1 illustrates the difference between PWM and sinus waveform signal. This results in torque oscillations over a wide frequency range and with large amplitudes, sometimes leading to the rupture of several rotating parts due to resonance [1]. The aim of this study is to suggest and compare rotors dynamic modeling able to capture the phenomenon.

In this paper, two models are submitted : the first one consists by coupling magnetic and mechanical finite elements models whereas the second one uses reduced order modelling to get rotor oscillations faster. The workflow of the methodologies can be found in the figure 2 .This study will allow to determine which scale of refinement is necessary to describe the problem with a good accuracy.

## 2 Design methodologies

### 2.1 Weakly coupled magnetic and mechanic Finite Elements Model

Firstly, we propose a methodology that couples magnetic and mechanic Finite Elements Models (FEM). The aim of this study is to evaluate the relevance of Maxwell mechanical stress projection on mechanical degrees of freedom (dof), see figure 3. The magnetic model is built with Flux 2D (Altair) whereas the mechanical one is set with Optistruct (Altair).

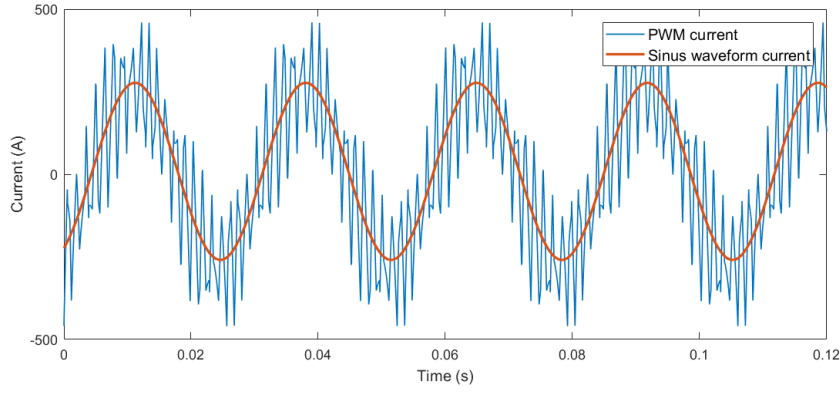


Figure 1: Current waveform with PWM or sinus

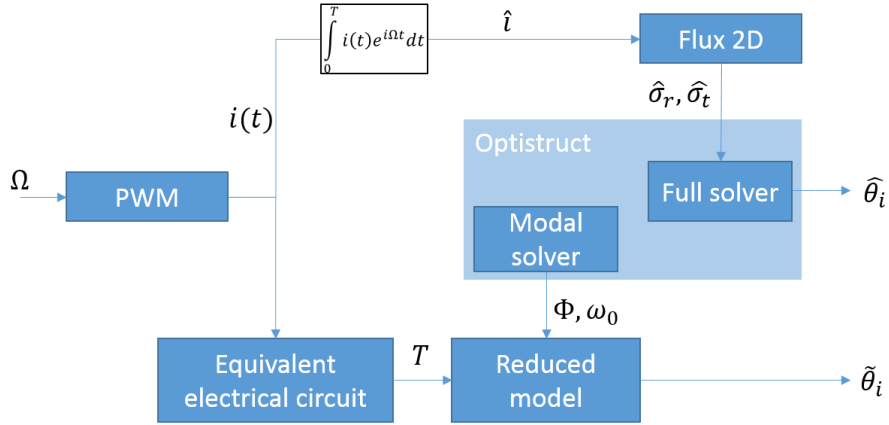


Figure 2: Workflow of methodologies for weakly coupled model and reduced model

Flux2D/Optistruct coupling is introduced in the thesis of Jaafar Hallal [2]. The goal is to develop a multi-physic model between 2D magnetic model and 3D mechanical model in order to evaluate the dynamic behavior coming from magnetic forces to study acoustic noise. Here, the results of this work are used to determine the vibratory fatigue failure that may occur.

The magnetic calculation takes place in two steps : firstly a magnetic harmonic response allows to initialize transient magnetic computation. It is mandatory to define geometry, materials, mesh, and then to create the electrical circuit that corresponds to the motor. Each component of the mesh is assigned to electrical component. Finally, we define the solving scenario: voltage, current, frequency, number of rotation, discretization angular steps. In our case, we simulate a working point that corresponds to a fixed rotating speed. The motor is powered by sinus waveform signal. The magnetic field is governed by the following set of equations.

$$\begin{aligned}
 \overrightarrow{\text{rot}}(\vec{E}) &= -\frac{\partial \vec{B}}{\partial t} \\
 \text{div}(\vec{B}) &= 0 \\
 \overrightarrow{\text{rot}}(\vec{H}) &= \vec{J} \\
 \vec{J} &= \sigma \vec{E} \\
 \vec{B} &= \mu \vec{H}
 \end{aligned} \tag{1}$$

where  $\vec{E}$  is the electrical field (V/m),  $\vec{B}$  is the magnetic induction (T),  $\vec{H}$  is the magnetic field (A/m),  $\vec{J}$  is the current density (A/m<sup>2</sup>),  $\sigma$  is the conductivity (S) and  $\mu$  is the permeability (H/m). Flux 2D solves the following equation including magnetic potential vector  $\vec{A}$  and electric potential  $V$ , see eqn. 2.

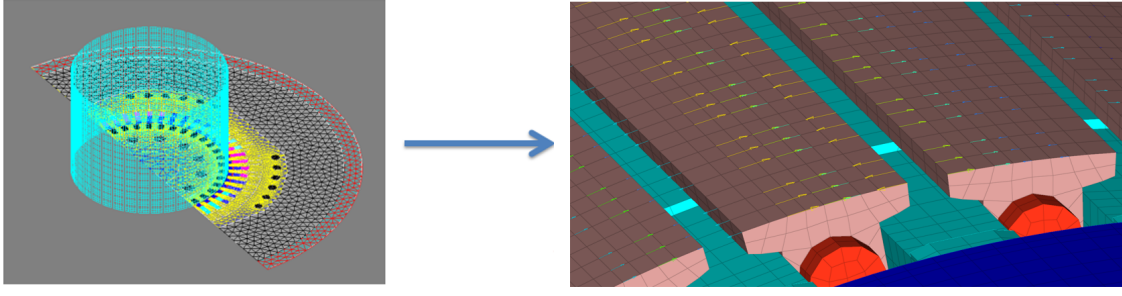


Figure 3: Maxwell pressure projection

$$\overrightarrow{rot} \left( v_0 [v_r] \overrightarrow{rot} (\vec{A}) - \vec{H}_c \right) + [\sigma] \left( \frac{\partial \vec{A}}{\partial t} + \overrightarrow{grad} (V) \right) = 0 \quad (2)$$

where  $[v_r]$  is the magnetic reluctivity,  $v_0$  is the vacuum reluctivity ( $v_0 = 1/\mu_0 = 1/(4\pi \cdot 10^{-7})$  m/H),  $\vec{A}$  is the magnetic potential vector (Wb/m),  $\vec{H}_c$  is the coercitive field (A/m),  $[\sigma]$  is the conductivity tensor (S) and  $V$  is the electric potential (V). The link between eqn. 2 and eqn. 1 is given by  $\vec{B} = \overrightarrow{rot} (\vec{A})$ . It allows to determine the magnetic induction  $\vec{B}$  and then the Maxwell pressure (3).

$$\begin{aligned} \sigma_r &= \frac{1}{\mu_0} (B_r^2 - B_t^2) \\ \sigma_t &= \frac{1}{\mu_0} B_r B_t \end{aligned} \quad (3)$$

where  $[B_r]$  is the radial magnetic induction (T),  $[B_t]$  is the tangential magnetic induction (T),  $[\sigma_r]$  is the radial pressure (N/m<sup>2</sup>) and  $[\sigma_t]$  is the tangential pressure (N/m<sup>2</sup>). The mechanical frequency response is driven by the following set of equations.

$$\begin{aligned} -\omega^2 \rho \hat{u} &= \text{div}(\hat{\sigma}) \\ \hat{\sigma} &= H \hat{\varepsilon} \\ \hat{\varepsilon} &= \frac{1}{2} (\text{grad}(\hat{u}) + \text{grad}^T(\hat{u})) \end{aligned} \quad (4)$$

Flux 2D generates loadings for the mechanical model:

$$\hat{\sigma} \cdot n = \begin{cases} \sigma_r \\ \sigma_{t_1} \\ \sigma_{t_2} \end{cases} \text{ on } \Gamma_l \quad (5)$$

where  $\Gamma_l$  is the set of rotor lamination surfaces. Both equations allow to know the displacement  $u$  and stress  $\sigma$  in the rotor as a function of the frequency. Rotations can be determined by using a transformation matrix  $C$  defined in the following relation :

$$\theta = Cu \quad (6)$$

with  $\theta$  the rotation of sections,  $C$  the transformation matrix built with speed composition relation and  $u$  the nodal displacement of the finite element model.

## 2.2 Mechanical reduced order model

An electromechanical lumped model has already been published by Bruzzese in 2016 [3]. He managed to define electro-technic model by solving equivalent electrical circuit and apply magnetic torque on a simplified mechanical model composed by masses, springs and dampers. Inertia and stiffness matrices were calculated by a lumped representation.

We offer to use a similar model to represent the electro-technical part (by solving equivalent equation of the motor), using d-q frame instead of  $\alpha$ - $\beta$  frame, but taking into account all parameters of the inverter and the train (inertial effect). Upstream, algorithms are used to generate PWM signal. Thus, this model allows us to simulate torque oscillations. Besides, we propose to compute mechanics from modal basis and nodal

coordinates of sections coming from the 3D FEM in order to have a more accurate mechanical representation of the real rotor.

The key of such a model is the transformation matrix that allows the reduction. The interest of model reduction is based on the dimension change of the problem. We start with a complete 3D model composed of thousands of translation degrees of freedom and we reduce it in order to obtain a model with only nine rotational degrees of freedom.

Then, we choose to divide the rotor in nine sections as described in figure (4).

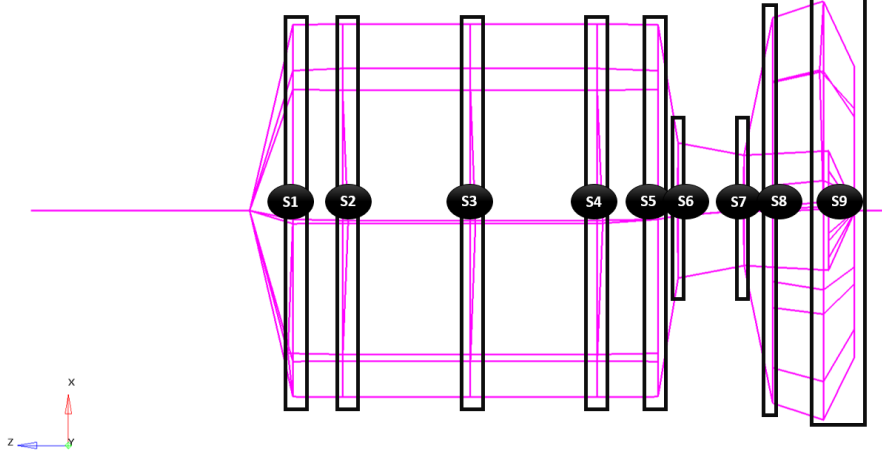


Figure 4: Rotor divided in 9 sections

Next, several matrices are built in order to create a transformation matrix. We define generalised coordinates  $q$  as the projection on the modal basis  $\Phi$  of the displacement  $u$ . Let's consider a transformation matrix  $A$  between the displacement  $u$  and the rotation of sections  $\theta$ , see eqn. 7.

$$\begin{aligned} u &= \Phi q \\ u &= A \theta \\ \Phi_{\theta} &= \begin{cases} A^{-1} \Phi \\ C \Phi \end{cases} \end{aligned} \quad (7)$$

where  $A$  is defined thanks to the speed composition relation and contains coordinates of nodes in sections ( $A$  is the inverse matrix of  $C$  defined in the previous paragraph) and with  $\Phi_{\theta}$  the modal basis expressed in rotation. The set of equations (7) gives:

$$q = (A \Phi_{\theta})^{-1} A \theta = P \theta \quad (8)$$

The classic dynamic equation in translation is given by the following equation.

$$[M] \ddot{u} + [D] \dot{u} + [K] u = F \quad (9)$$

Writted on the modal basis, the dynamic equation in generalized coordinates is :

$$\ddot{q} + [\Delta] \dot{q} + [\Lambda] q = f \quad (10)$$

with  $[\Delta] = 2\xi \omega_0 [I]$ .

By using the transformation (equation 8), the equation in the rotation coordinate frame becomes :

$$[J] \ddot{\theta} + [D_{\theta}] \dot{\theta} + [K_{\theta}] \theta = T, \quad (11)$$

with

$$\begin{cases} [J] = P^T P, \\ [K_{\theta}] = P^T [\Lambda] P, \\ [D_{\theta}] = P^T (2\xi \omega_0 [I]) P. \end{cases} \quad (12)$$

Rotations of rotor sections are obtained by solving the equation (11).

### 3 Comparison of results

Rotations of rotor sections are obtained with both methods presented above. This paragraph shows the results on the torsion modes, see figure 5. The first table illustrates the validation of the model reduction with PWM excitation, whereas the second table compares the coupled model with sine wave excitation and the reduced model with PWM excitation.

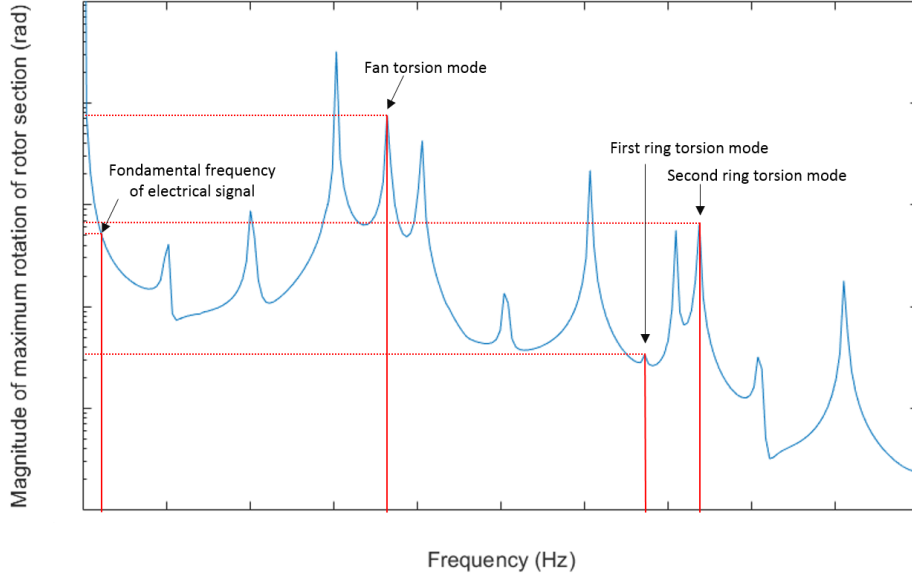


Figure 5: Frequency Response Function

	PWM + FEM	PWM + Reduced model	Relative error (%)
Fondamental frequency	6.648E-7	6.645E-7	0.05
Fan torsion mode	7.03E-6	7.591E-6	7.98
First ring torsion mode	3.363E-8	3.374E-8	0.33
Second ring torsion mode	7.269E-7	6.44E-7	11.4

Table 1: Comparison of results obtained with both models and PWM input - Maximal rotation (rad)

Values of the table 1 are close and permit to justify the validation of the model reduction.

	Sine wave + FEM	PWM + Reduced model	Relative error (%)
Fondamental frequency	1.134E-6	6.645E-7	41.4
Fan torsion mode	1.504E-7	7.591E-6	4947.2
First ring torsion mode	7.779E-9	3.374E-8	333.7
Second ring torsion mode	7.084E-8	6.44E-7	809

Table 2: Comparison of results obtained with both models with sine wave or PWM input - Maximal rotation (rad)

Table 2 shows the difference between the weakly coupled model with sine wave input and reduced model with PWM input. Values are not comparable except for the fondamental frequency of the PWM that corresponds to the sine wave frequency. This illustrates the necessity to use the coupled model to simulate the working point with PWM excitation to validate the approach.

The solving of the equivalent circuit is immediate, the computation of modal basis takes 8 minutes on a calculator with 16 CPU, 3.2GHz and 225692 MB RAM and then the calculation of rotation thanks to reduced model takes 5 seconds. In comparison, the magnetic computation with Flux 2D takes 9 hours in a laptop with 8 CPU, 2.7GHz and 16 Go RAM and then the mechanical computation with Optistruct takes 2 hours on the

calculator. The model reduction permits to pass from 11 hours to 8 minutes of computation with in addition PWM excitation.

## 4 Conclusion

This paper propose two methods to model the problem of vibratory fatigue that happened on train rotor subjected to PWM excitation. The first one combines two Finite Elements Models in order to solve the multiphysic problem. The projection of magnetic excitation on mechanical mesh allows to determine rotation of rotor sections. Secondly, we propose a model reduction to improve the computation time during design phase. Transformation matrix is proposed to reduce model dimension and also to change the observation in rotation.

As mentioned, the Flux 2D simulation were performed with a sinus waveform instead of PWM, whereas the reduce model included PWM excitations. Values are not comparable. The next step could be to simulate the problem with PWM. Other points to improve are to simulate variable speed and to get stress with both models instead of rotations to be able to design rotor in vibratory fatigue.

## References

- [1] Bruzzese, Tassarolo, and Santini, "Failure Root-Cause Analysis of End-Ring Torsional Resonances and Bar Breakages in Fabricated-Ca," 2016.
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- [3] Bruzzese, "Electromechanical modeling of a railway induction drive prone to cage vibration failures," in *IECON 2016 - 42nd Annual Conference of the IEEE Industrial Electronics Society*, IEEE, 2016.