Squared envelope analysis based on the H_∞ filter order tracking: Application for bearing diagnosis

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Abstract

The analysis of the squared envelope spectrum (SES) is one among the most used tools for bearing diagnosis. It can easily reveals the characteristic frequencies related to the bearing fault [1, 2]. Actually, the envelope is estimated through a demodulation process in a selected frequency band. The proper choice of the latter is really challenging in a complex environment [3]. In addition to that, the frequency of the bearing fault is likely to be masked by deterministic components. This can jeopardize the efficiency of classical techniques [3, 4, 5]. In this paper, a new approach for bearing diagnostic is proposed. It is based on a recently proposed order tracking technique using the H_∞ filter [7]. In details, the method starts by computing the squared envelope (SE) of the raw signal over the full demodulation band without prior processing. Next, the SE is modeled in a state space using a trigonometric series expansion. Last, an H_∞ estimator is designed to extract the amplitude of each harmonic related to the bearing fault signature. This estimator is well convenient to track the order of bearing faults, particularly in the presence of deterministic components (i.e. the noise). Since this noise is neither white nor Gaussian, the traditional Kalman filter order tracking is compromised [8, 9, 10]. Contrary to the Kalman filter, the H_∞ filter is based on the minimax optimization. The minimax approach leads to the minimization of the estimation error for the worst possible amplification of the noise signal. More interestingly, no prior knowledge about the statistical properties of the noise signals is required [11, 12]. The efficiency of the proposed approach is demonstrated on simulated and real-world vibration signals in nonstationary regimes.

Keywords: H_∞ filter, state space modelling, order tracking, squared envelope, bearing diagnosis, vibration signal, variable speed condition.

1 Introduction

Rolling element bearings are among the most widely used elements in rotating machines. Because of their common role to carry high loads, bearings are likely to be exposed to sudden failures causing system outage. Thus, there has been an increasing interest in developing appropriate techniques for signal denoising and incipient fault detection. Due to their non-invasive nature and their high reactivity to incipient faults, the development of vibration-based techniques has spiked the interest of the scientific community [1]. In this context, envelope analysis has long been recognized as a powerful bearing diagnosis technique. Typically, it consists of a bandpass filtering step in a frequency band wherein the impulsive response is amplified, followed by a demodulation that extracts the signal envelope. The spectrum of the envelope reveals the desired diagnostic information, including the repetition frequency of the fault as well as possible modulations. It has been shown in [2] that it is preferable to use the squared envelope instead of the envelope as the latter is likely to introduce additional interfering components in the envelope spectrum. Since that time, the envelope spectrum was replaced by the squared envelope spectrum (SES) which has become the benchmark technique for bearing diagnostics. A powerful solution to this issue was proposed through the spectral kurtosis [16] (and some derived tools such as the kurtogram [17], the fast kurtogram [3], etc.) which provides an entirely blind way of identifying the best demodulation band according to the impulsivity criterion. Despite its remarkable relevance in machine

signal analysis, the efficiency of the spectral kurtosis is compromised in many situations; for instance, in the presence of energetic deterministic part or the presence of multiple impulsive sources or strongly in nonstationary conditions. This paper comes in this context aiming at providing a new way to address bearing diagnostic based on tracking bearing characteristic orders (i.e. fault frequencies referenced to the shaft frequency) in the squared envelope, without the need of eliminating the deterministic component neither to filter the signal. The method uses the fact that the SE signal comprises a cyclic patterns related to bearing fault. From this observation, the SE signal is described in the state space model using a trigonometric series expansion. Then, an H_∞ filter is designed to track bearing fault order components. This approach is different from the classical Kalman filter based order tracking. The latter is a widespread method used to track sinusoidal components [8, 9, 10], assuming that the exegeneous noises that affect the state model are white and Gaussian with known statistics. In current situations, those assumptions are not valid. Indeed, the meshing components that mask the bearing ones are neither white nor Gaussian. To address this issue, an H_∞ filter is proposed. The latter minimizes the estimation error for the worst possible amplification of the noises. This leads to a minimax optimization where no prior knowledge about the statistical properties of the noises is required [11, 12].
The proposed approach is presented in this paper as follows. In Section 2, the SE of a discrete vibration signal is described in the state space model using a trigonometric series expansion. In Section 3, the methodology to design an H_∞ filter is exposed. In Section 4, the proposed approach is first applied to a simulated vibration signal. Then, it is applied to analyze real-life vibration signals acquired from a wind turbine under nonstationary conditions. Conclusions of this paper are given in Section 5.

2 State space modeling of the squared envelope signal

Condider the discrete measured vibration signal as follows:

$$y[k] = y_r[k] + y_g[k] + b[k] \tag{1}$$

where $y_r[k]$ is the signal related to the bearing vibration, $y_g[k]$ is the meshing signal and $b[k]$ is the signal composed of all the exegeneous vibrations such as the background noise for all $k = 1, \cdots, N$. N is the number of signal samples. The meshing signal, in the case of a tooth crack, exhibits amplitude and phase modulations [6]. The corresponding signal in nonstationary regimes can be written as:

$$y_g[k] = \kappa(\omega[k]) \sum_m a_m[k] e^{j\phi_m[k]} e^{j\theta_m[k]} \tag{2}$$

in which $\kappa(\omega[k])$ is a modulation function depending on the machine regime, $\omega[k] = 2\pi f_r[k]$ is the shaft angular speed and f_r is the machine rotating frequency, a_m and ϕ_m are respectively the amplitude and phase modulations, θ_m is the instantaneous meshing angular displacement and j is the complex number such as $j^2 = -1$. Concerning the bearing vibration signal, it exhibits a series of impulses which can be modelled as [1]:

$$y_r[k] = \kappa(\omega[k]) M[k] \sum_i^d A_i I[k - \lceil T_i f_s \rceil] \tag{3}$$

in which:

- $M[k]$ is the load distribution function for an inner-race under radial load. In stationary conditions, this function is periodic at the shaft rotating period [18];
- A_i is the amplitude of the ith impact so that $A_i = A + \delta A_i$. A is the mean value of the distribution and δA_i is a zero-mean random part with σ_A its standard deviation;
- T_i is the instant of apparition of the ith impact;
- I is the damping response that depends on the damping factor and the resonance frequency of the bearing structure;
- d is the number of impacts resulting from the bearing fault;

- f_s is the sampling frequency;
- $\lceil\cdot\rceil$ stands for the integer part of a decimal number.

Since the bearing's rolling elements are subject to slipping phenomena, the time of occurence from one impact to another is not constant. This time exhibits a random part and, as mentionned in [1], can be modelled in stationary conditions as:

$$T_i = iT + \delta T_i \tag{4}$$

where T is the time isntant between two consecutive impacts and δT_i is a random variable with a Gaussian distribution. This modelling is no longer valid in nonstationary conditions. In this context, Borghesani et *al.* [19] and Abboud et *al.* [20] have written the instant of impact occurrence as follows:

$$T_i = t(i\theta_d + \delta\theta_i) \tag{5}$$

in which θ_d is the angular period of the bearing fault and $\delta\theta_i$ is a zero-mean Gaussian distribution.
The squared envelope of the measured vibration signal, which is of interest in this work and denoted SE, is given as follows:

$$\begin{aligned} \mathrm{SE}[k] &= \mathbb{E}\{y[k]\bar{y}[k]\} && (6)\\ &= \mathbb{E}\{(y_r[k]+y_g[k]+b[k])(\bar{y}_r[k]+\bar{y}_g[k]+\bar{b}[k])\} && (7)\end{aligned}$$

where $\bar{a}$ is the conjugate of the complex number a and $\mathbb{E}\{\cdot\}$ stands for the expectation symbol.
In this paper, it is assumed that the bearing, the meshing and the noise signals are mutually not correlated. Hence, the squared envelope becomes:

$$\begin{aligned} \mathrm{SE}[k] &= \mathbb{E}\{y_r[k]\bar{y}_r[k]\}+\mathbb{E}\{y_g[k]\bar{y}_g[k]\}+\mathbb{E}\{b[k]\bar{b}[k]\} && (8)\\ &= \mathbb{E}\{y_r[k]\bar{y}_r[k]\}+n[k] && (9)\end{aligned}$$

where $n[k] = \mathbb{E}\{y_g[k]\bar{y}_g[k]\}+\mathbb{E}\{b[k]\bar{b}[k]\}$ is considered as a noise signal. Otherwise, the SE can be expressed using the autocorrelation function (ACF) denoted by $\mathscr{R}$. The latter, applied to the bearing signal y_r in equation (3), can be written as [20]:

$$\mathscr{R}[k,j] \approx (A^2+\sigma_A^2)\kappa^2(\omega[k])M^2[k]\sum_{i=1}^{d}\mathbb{E}\{g[k-\lceil T_i f_s\rceil, j]\} \tag{10}$$

where $g[k,j] = I[k]I[k-j]$. By writing this function in the angular domain, one gets:

$$\mathscr{R}[k_a,j] \approx (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\tilde{M}^2[k_a]\sum_{i=1}^{d}\mathbb{E}\{\tilde{g}[k_a-\lceil\theta_i N_a\rceil, j]\} \tag{11}$$

with $\tilde{x}$ the angular transformation of the time variable x, k_a the sample index in the angular domain, Na the angular sampling frequency and θ_i the angle instant of the ith impact occurrence. Refering to equation (5), the latter is modelled as $\theta_i = i\theta_d + \delta\theta_i$ [22]. Thus, the ACF becomes:

$$\mathscr{R}[k_a,j] \approx (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\tilde{M}^2[k_a]\sum_{i=1}^{d}\mathbb{E}\{\tilde{g}[k_a-k_{a,i}-\delta k_{a,i}, j]\} \tag{12}$$

where $k_{a,i} \approx \lceil i\theta_d N_a\rceil$ is the angular sample of the ith impact occurrence and $\delta k_{a,i} \approx \lceil\delta\theta_i N_a\rceil$ is a random integer. The above equation of ACF has been proven by Abboud et *al.* in [20]. By taking advantage of this equation, the SE, using the ACF, can be expressed as:

$$\begin{aligned} \mathrm{SE}[k_a] &= \mathscr{R}[k_a, j=0]+\tilde{n}[k_a] && (13)\\ &= (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\tilde{M}^2[k_a]\sum_{i=1}^{d}\mathbb{E}\{\tilde{g}[k_a-k_{a,i}-\delta k_{a,i}, j=0]\}+\tilde{n}[k_a] && (14)\\ &= (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\tilde{M}^2[k_a]\sum_{i=1}^{d}\mathbb{E}\{\tilde{h}^2[k_a-k_{a,i}-\delta k_{a,i}]\}+\tilde{n}[k_a] && (15)\end{aligned}$$

Assume that the random variable $k_a - k_{a,i} - \delta k_{a,i}$ has a probability density function $f[\delta k_{a,i}]$ centered at $k_a - k_{a,i}$ with a constant standard deviation. According to the *law of the unconscious statistician* [15], the above equation is written as:

$$\mathrm{SE}[k_a] = (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\tilde{M}^2[k_a]\sum_{i=1}^{d}\sum_{i=1}^{d}\tilde{h}^2[k_a-k_{a,i}-\delta k_{a,i}]f[\delta k_{a,i}]+\tilde{n}[k_a] \tag{16}$$

$$= (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\tilde{M}^2[k_a]\sum_{i=1}^{d}(\tilde{h}^2\circledast f)[k_a-k_{a,i}]+\tilde{n}[k_a] \tag{17}$$

$$\mathrm{SE}[k_a] = (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\tilde{M}^2[k_a]\sum_{i=1}^{d}s[k_a-k_{a,i}]+\tilde{n}[k_a] \tag{18}$$

in which $\circledast$ stands for the convolution symbol and $s[k_a] = (\tilde{h}^2 \circledast f)[k_a]$ is the convolution between the function $\tilde{h}^2$ and f. The function $\tilde{M}^2[k_a]$ is deterministic and can be approximated by a Fourier series such as $\tilde{M}^2[k_a] = \sum_s \lambda_s[k_a]e^{j\psi_s[k_a]}e^{js\theta_r[k_a]}$ where λ_s and ψ_s are respectively the sth variable amplitude and phase of the Fourier series and θ_r is the angular period of the shaft. In same way, the sum in the SE formula can also be expressed by $\sum_{i=1}^{d} s[k_a-k_{a,i}] = \sum_z \rho_z[k_a]e^{j\varphi_z[k_a]}e^{jz\theta_d[k_a]}$ with ρ_z and φ_z respectively the zth variable amplitude and phase of the Fourier series and θ_d the angular period of bearing fault. This leads to:

$$\mathrm{SE}[k_a] = (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\Re\{\sum_{s,z}\lambda_s[k_a]\rho_z[k_a]e^{j(\psi_s[k_a]+\varphi_z[k_a])}e^{j(z\theta_d[k_a]\pm s\theta_r[k_a])}\}+\tilde{n}[k_a] \tag{19}$$

where $\Re\{.\}$ defines the real part of a complex number. In this paper, all the components related to the shaft angular period are not of interest. Therefore, the SE is written as:

$$\mathrm{SE}[k_a] = (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\Re\{\sum_{s=0,z}\lambda_0[k_a]\rho_z[k_a]e^{j(\psi_0[k_a]+\varphi_z[k_a])}e^{j(z\theta_d[k_a])}\}+v[k_a] \tag{20}$$

$$\approx \sum_{z=1}^{l}\alpha_z[k_a]\cos(z\theta_d[k_a]+\phi_z[k_a])+v[k_a] \tag{21}$$

in which $\alpha_z[k_a] = (A^2+\sigma_A^2)\tilde{\kappa}^2(\omega[k_a])\lambda_0[k_a]\rho_z[k_a]$, $\phi_z[k_a] = \psi_0[k_a]+\varphi_z[k_a]$ are respectively the zth amplitude and phase of the Fourier series, $v[k_a]$ is the noise comprising the initial noise $\tilde{n}[k_a]$ and all the components related to the shaft angular period $\theta_r[k_a]$ and l is the higher order of the series. The latter defines the number of the bearing component of interest in the estimation procedure. In the state modelling approach that is proposed in this paper, $v[k_a]$ is the so-called *measurement error* or *measurement noise*.

At this stage, the detection of the bearing fault is reduced to the estimation of the amplitude α_z and the phase ϕ_z of the zth order component. This estimation can be done using a linear or a non-linear filtering approach. However, the non-linear approach are subject to a divergence issue. To obtain a linear model, the SE signal is presented in the following form:

$$\mathrm{SE}[k_a] = \sum_{z=1}^{l}\boldsymbol{h}_z^T[k_a]\boldsymbol{x}_z[k_a]+v[k_a] \tag{22}$$

where:

- $\boldsymbol{h}_z[k_a] = \begin{pmatrix} \cos(z\theta_d[k_a]) & \sin(z\theta_d[k_a]) \end{pmatrix}^T \in \mathbb{R}^{2\times 1}$ is the zth measurement vector. $\mathbb{R}$ stands for the ensemble of the real number and $(\cdot)^T$ for the transpose symbol. In the rest of this paper, the lowercase symbols in bold stand for vectors and the uppercase ones in bold stand for matrices;

- $\boldsymbol{x}_z[k_a] = \begin{pmatrix} \alpha_z[k_a]\cos(\phi_z[k_a]) & \alpha_z[k_a]\sin(\phi_z[k_a]) \end{pmatrix}^T \in \mathbb{R}^{2\times 1}$ is the zth state variable.

Using equation (22), the estimation of the amplitude and the phase of the Fourier series reduces to the estimation of the state variable $\boldsymbol{x}_z$. In this context, the simultaneous estimation of $\boldsymbol{x}_z$ leads to write the SE as:

$$\mathrm{SE}[k_a] = \boldsymbol{h}^T[k_a]\boldsymbol{x}[k_a]+v[k_a] \tag{23}$$

in which $\boldsymbol{h}[k_a] = \begin{pmatrix} \boldsymbol{h}_1^T[k_a] & \cdots & \boldsymbol{h}_l^T[k_a] \end{pmatrix}^T \in \mathbb{R}^{2l\times 1}$ and $\boldsymbol{x}[k_a] = \begin{pmatrix} \boldsymbol{x}_1[k_a] & \cdots & \boldsymbol{x}_l[k_a] \end{pmatrix}^T \in \mathbb{R}^{2l\times 1}$ are respectively the *measurement vector* and the *state variable*. Assuming that the angular period of the bearing fault is known, the detection of the latter is reduced to the estimation of the state variable $\boldsymbol{x}[k_a]$. In this context, it is proposed to estimate $\boldsymbol{x}[k_a]$ in a recursive manner using a state space modelling approach.
For this reason, the SE signal is described in a state space. That is to say the dynamic of the state variable has to be defined. In this paper, all parameters include in the state variable $\boldsymbol{x}[k_a]$ are supposed to follow, roughly speaking, a *random walk* so that:

$$\boldsymbol{x}[k_a+1] = \boldsymbol{x}[k_a] + \boldsymbol{w}[k_a] \tag{24}$$

where $\boldsymbol{w}[k_a]$ is a random or deterministic signal with bounded energy. Equations (23) and (24) form the state space model of the *SE* signal. From the latter, an H_∞ filter is designed in the next section for the state variable estimation.

3 H_∞ filter order tracking

Considering equations (23) and (24), the H_∞ filter will be designed to estimate some arbitrary linear combination of the state, say:

$$\hat{s}[k_a] = \boldsymbol{h}^T[k_a]\hat{\boldsymbol{x}}[k_a] \tag{25}$$

where $\hat{x}[k_a]$ satisfies the following recursion:

$$\hat{\boldsymbol{x}}[k_a] = \hat{\boldsymbol{x}}[k_a-1] + \boldsymbol{g}[k_a]\left(\mathrm{SE}[k_a-1] - \boldsymbol{h}^T[k_a-1]\hat{\boldsymbol{x}}[k_a-1]\right) \tag{26}$$

where $\boldsymbol{g}[k_a]$ is the H_∞ gain and $\hat{\boldsymbol{x}}[k_a]$ is the estimate of $\boldsymbol{x}[k_a]$. The state variable is estimated for any $v[k_a]$ and $\boldsymbol{w}[k_a]$ of bounded energy.
Let $e[k_a] = s[k_a] - \hat{s}[k_a]$ be the estimation error, then the H_∞ gain is found by minimizing the following cost function given by [12]:

$$J = \frac{\sum_{k_a=1}^{N} \|e[k_a]\|^2}{\|e[1]\|^2_{\boldsymbol{P}[1]^{-1}} + \sum_{k_a=1}^{N}\left(\|\boldsymbol{w}[k_a]\|^2_{\boldsymbol{Q}^{-1}} + \|v[k_a]\|^2_{R^{-1}}\right)} \tag{27}$$

where $(e[1], \boldsymbol{w}[k_a], v[k_a]) \neq (0,0,0)$, $e[1]$ represents the initial error, $\boldsymbol{P}[1] > 0$, $\boldsymbol{Q} > 0$ and $R > 0$ are positive definite weighting matrices, N is the number of samples and $\|e[k_a]\|_{\boldsymbol{S}} = e[k_a]^T Se[k_a]$. This can be interpreted as the energy gain from the unknown disturbances $\boldsymbol{P}^{-1/2}[1]e[1]$ and $\{\boldsymbol{Q}^{-1/2}\boldsymbol{w}[k_a], R^{-1/2}v[k_a]\}_{k_a=1}^{N}$ to the estimation error $\{e[k_a]\}_{k_a=1}^{N}$. It is quite clear that if the ratio in (27) is small then the estimation is better, and vice versa. However, this ratio depends on the quantities $e[1]$, $\boldsymbol{w}[k_a]$ and $v[k_a]$ which are unknown. In this context, the worst case is considered below:

$$\underset{e[1],\boldsymbol{w}[k_a],v[k_a]}{\mathrm{Sup}} \quad J \leq 1/\gamma \tag{28}$$

where "Sup" stands for the supremum and γ is the performance bound. Otherwise, the goal of the H_∞ problem is to find an estimation $\{\hat{s}[k_a]\}_{k_a=1}^{N}$ that minimizes the worst-case energy. This is equivalent to minimize the following scalar quadratic form:

$$J_f = e^T[1]\boldsymbol{P}^{-1}[1]e[1] + \sum_{k_a=1}^{N} \boldsymbol{w}[k_a]^T\boldsymbol{Q}^{-1}\boldsymbol{w}[k_a] + \sum_{k_a=1}^{N} v[k_a]^T R^{-1} v[k_a] - \gamma \sum_{k_a=1}^{N} e[k_a]^T e[k_a] \tag{29}$$

so that $J_f > 0$ for all vectors $e[1]$, for all nonzero signals $\boldsymbol{w}[k_a]$ and $v[k_a]$ of bounded energy.
Giving the cost function J_f, the worst case minimization is reduces to minimize J_f in respect to $\hat{s}[k_a]$ and to maximize J_f in respect to $e[1]$, $\boldsymbol{w}[k_a]$ and $v[k_a]$ for all $k_a = 1, \cdots, N$. This leads to a minmax optimization formulated in such a way that:

$$\{\hat{s}[k_a]\}_{k_a=1}^{N} = \arg\left(\min_{\hat{s}} \max_{e[1],\boldsymbol{w},v} (J_f)\right) \tag{30}$$

This optimization problem can be solved by the well known Lagrange mutliplier approach. The solution to the above optimization is given by the theorem quoted below [11, 12, 14].

Theorem 1 *Let $\gamma > 0$ be the user-specified performance bound. Then, there exists an H_∞ estimation for $s[k_a]$ if and only if there exists a symmetric positive definite matrix $\boldsymbol{P}[k_a] \in \mathbb{R}^{2l \times 2l}$ that satisfies the following discrete-time Riccati equation:*

$$\boldsymbol{P}[k_a] = \boldsymbol{P}[k_a - 1]\boldsymbol{\Gamma}[k_a - 1] + \boldsymbol{Q} \tag{31}$$

when

$$\boldsymbol{\Gamma}[k_a] = \left(\boldsymbol{I}_{2l} - \gamma \boldsymbol{h}^T[k_a]\boldsymbol{h}[k_a]\boldsymbol{P}[k_a] + \boldsymbol{h}[k_a]R^{-1}\boldsymbol{h}^T[k_a]\boldsymbol{P}[k_a]\right)^{-1} \tag{32}$$

and $\boldsymbol{I}_{2l} \in \mathbb{R}^{2l \times 2l}$ is the identity matrix.
Then, the H_∞ gain $\boldsymbol{g}[k_a] \in \mathbb{R}^{2l \times 1}$ is given by:

$$\boldsymbol{g}[k_a] = \boldsymbol{P}[k_a]\boldsymbol{\Gamma}[k_a - 1]\boldsymbol{h}[k_a]R^{-1} \tag{33}$$

It should be noted that for some weighting matrices $\boldsymbol{P}[1]$, $\boldsymbol{Q}$ and R the performance criterion in (28) is achieved if and only if the performance bound γ satisfies the following inequality:

$$\gamma < R^{-1} \tag{34}$$

Since γ defines the noise level attenuation or the performance bound, it should be as high as possible. And, it has been shown in a previous paper [7] that when γ is greater than its optimal value, the matrix $\boldsymbol{P}$ is not symmetric positive definite. Otherwise, when γ tends to zero, the H_∞ filter is not constrainted. Then, it is equivalent to the standard Kalman filter for which R and $\boldsymbol{Q}$ are defined respectively as the covariance matrix of the measurement noise and the state noise.

4 Application

4.1 Synthetic signal analysis

Here, a synthetic signal is presented to evaluate the performance of the proposed approach in estimating bearing order components. The signal is composed of the bearing, the meshing and the noise signal. They represent respectively 20%, 30% and 50% of the synthetic signal energy. The non-stationary condition is simulated using a non-linear rotating frequency varying between 5 Hz and 30 Hz such as $f_r[k] = 5 + 25\sin\left(\frac{\pi(k-1)}{2(N-1)}\right)$ for $1 \leq k \leq N$ where N is the number of samples. The time duration of the signal is 5 s.
Concerning the meshing signal, sampled at the frequency $f_s = 10$ kHz, it is computed using equation (2) and contains five meshing components. The latter is composed of the 45th and 49th shaft order component with amplitude and phase modulations. The generation of these modulations is presented in details in Appendix A. About the bearing signal, it is generated using equation (3) in which the resonance frequency and the damping factor are respectively equal to 4 kHz and 2000.

The rolling bearing considered is subject to a local defect occurring on the outer-race. Its characteristic order, ball-pass-order on the outer-race denoted BPOO, is equal 8.7 times the rotating frequency with a slight random variation. Last, the additive noise is generated using a white Gaussian noise modulated by the rotating frequency.
The different contributions of the signal are displayed in Figure 1. It can be seen in the SES of the raw signal (Figure 1 (g)) that the bearing fault order is masked by deterministic components. Thus, the proposed approach is applied to track the order component of the bearing fault. The number of order harmonic of interest l is equal to 15. Also, the parameters of the filter take the following values: $R = 1$, $\boldsymbol{Q} = 0 \times \boldsymbol{I}_{2l}$, $\gamma = 0.9R^{-1}$ and the filter initial values are $\boldsymbol{P}[1] = \boldsymbol{I}_{2l}$ and $\boldsymbol{x}[1] = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{2l \times 1}$. The choice of $\boldsymbol{Q}$ is motivated by the fact the coefficients of the trigonometric series expansion are assumed to be constant. This means that the state noise $\boldsymbol{w}[k]$ is null for all $k = 1, \cdots, N$.
The result of the estimation provided by the H_∞ estimator for the choosen parameters is presented on Figure 2. There, the number of peaks estimated by the proposed approach correponds to the expected ones, i.e 15 peaks. Moreover, the deterministic components present in the SES of the raw signal have been greatly attenuated. Besides of that, some peaks, with lower energy level, appear around the bearing order components. This is due to the fact that the H_∞ filter, like all type of filter, don't atteanuate uniformely all the frequencies outside the

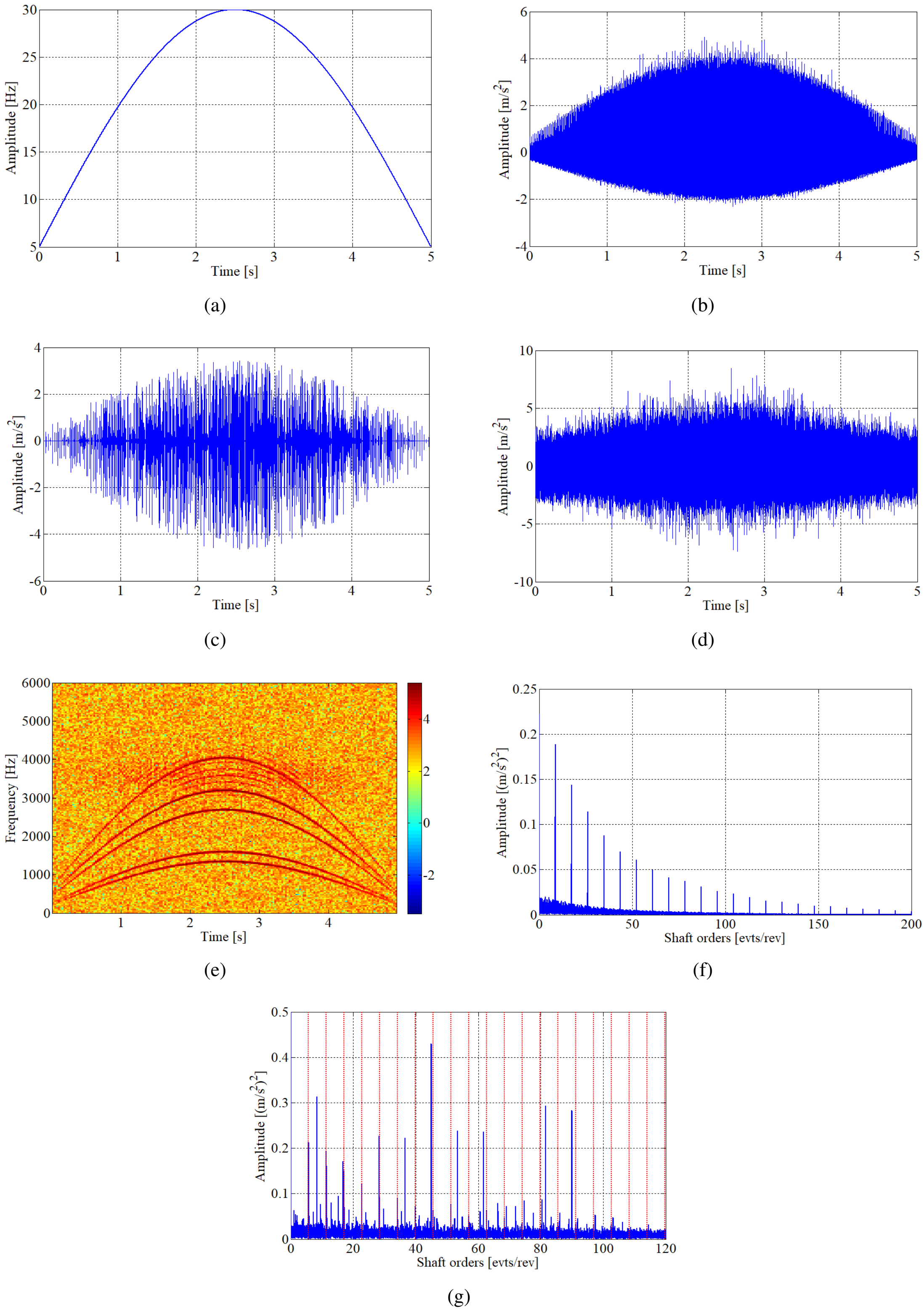


Figure 1 – Synthetic vibration signal: (a) Rotating frequency, (b) Meshing signal, (c) Bearing signal, (d) Complete vibration signal, (e) Time-frequency representation, (f) SES of the bearing signal and (g) SES of the complete signal.

band of interest.
To evaluate the performance of the proposed approach, the signal-to-noise ratio is calculated by the following formula $\text{snr} = 10 \times \log_{10}\left(\frac{\sum_{k=1}^{N} s^2[k]}{\sum_{k=1}^{N}(\hat{s}[k]-s[k])^2}\right)$ where s is the SE related only to the bearing signal and $\hat{s}$ is its estimate. When the bearing signal is totally influenced by the meshing and the noise signal, that is the worst case estimation, the estimation error is $\hat{s}[k] - s[k] = b[k]$. In this case, the snr is equal to -6 dB and defines the lower limit of the performance bound. On this basis, it can be stated that all estimations provided by the proposed approach should have a snr greater than -6 dB. It follows that a higher snr leads to a better estimation. For this first simulation, after a 200 Monte-Carlo simulations, the estimation error leads to a snr equal to 1.48 dB. This value is greater than the performance bound and corroborates the quality of the estimation displayed on Figure 2.

Influence of the fault frequency incertitude

In the results presented above the frequency (or order) of the bearing was exactly known. In real situations, this frequency is known with uncertainty. This can be due (i) to the fluctuation of the shaft rotating frequency, (ii) to the imperfection of the speed sensor or (iii) to the sliping phenomena of the bearing rolling's elements. In this section, the influence of the uncertainty of the bearing fault order on the estimation quality is investigated. Thus, the snr is evaluated for different values of uncertainty on the bearing fault order. This uncertainty varies from 0 % to 5 % of the real value of the order of the bearing fault. The snr obtained in the range of the uncertainty is displayed on Figure 3. There, the snr remains constant when the uncertainty varies from 0% to 2.5%. This is interesting for the bearing health monitoring since the uncertainty on a potential fault frequency can reach 2% in real situations as mentionned by Randall and Antoni [1]. Beyond this value, the snr decreases; thus the quality of the estimation is degraded and the proposed approach is no longer robust to track the bearing order components.

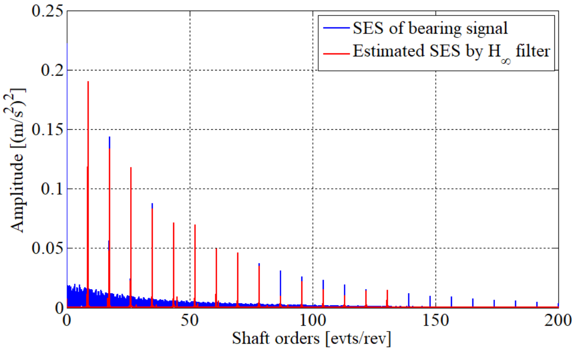


Figure 2 – Estimation of the SES of the bearing signal provided by the H_∞ filter. Here, the uncertainty on the bearing fault order is equal to zero.

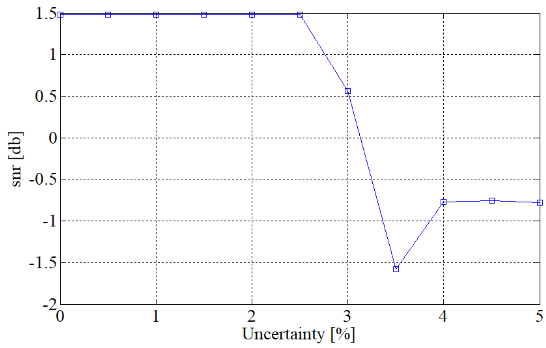


Figure 3 – Signal-to-noise ratio (snr) evolution against the uncertainty on the bearing fault order.

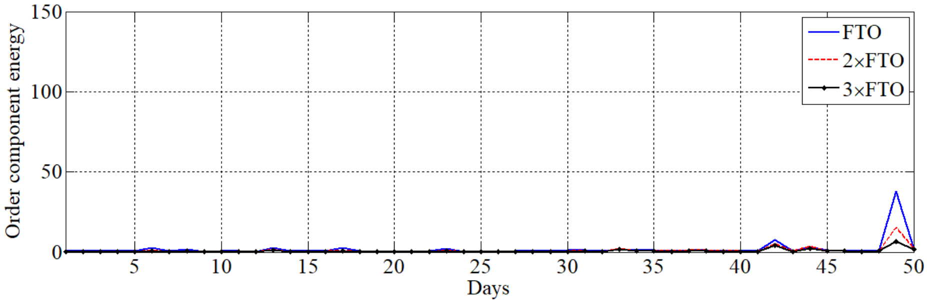
4.2 Experimental signal analysis

This subsection deals with signals acquired from a 2 megawatts wind turbine high speed shaft on which a condition monitoring system is installed. The bearing has an inner race fault which is increasing in severity across the 50-day period. At the end of the test, the bearing was inspected and a crack has been identified in the inner race. Acceleration signals were recorded on a daily basis (one signal per day) together with tachometer signals, over a 6 s duration each with a sampling frequency equal to 97656 Hz. The nominal speed of the bearing shaft is 1800 rpm (30 Hz). Note that the speed variability has reached 15 % of the nominal speed in some records; the reason why the regime is considered nonstationary. The theoretical fault frequencies referenced to the shaft frequency are as follows:

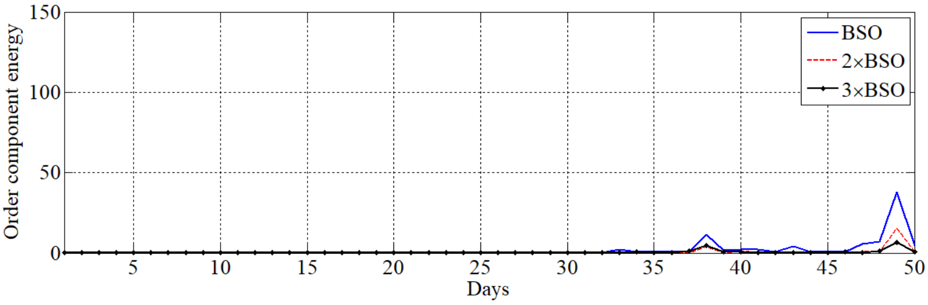
- Ball pass order on outer-race: BPOO = 6.72;
- Ball pass order on inner-race: BPOI = 9.47;

- Ball spin order: BSO $= 1.435$;
- Fundamental train order: FTO $= 0.42$.

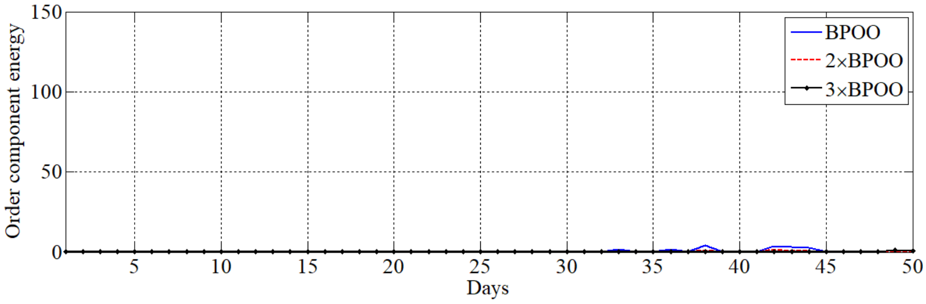
More information can be found in [21]. In this section, the proposed approch is applied to track the order components locolated at $i \times$ BPOO, $i \times$ BPOI, $i \times$ BSO and $i \times$ FTO where $i = 1, 2, 3$. The parameters of the H_∞ filter take the following values: $R = 1$, $\boldsymbol{Q} = 0 \times \boldsymbol{I}_3$ and $\gamma = 0.95$ and the filter initial values are $\boldsymbol{P}[1] = \boldsymbol{I}_3$ and $\boldsymbol{x}[1] = \begin{bmatrix} 1 & \cdots & 1 \end{bmatrix} \in \mathbb{R}^{6 \times 1}$. To monitor the health state of each component of the bearing during the 50-day periods, the energy of each order component is evaluated. Figure 4 shows the evolution of this energy during



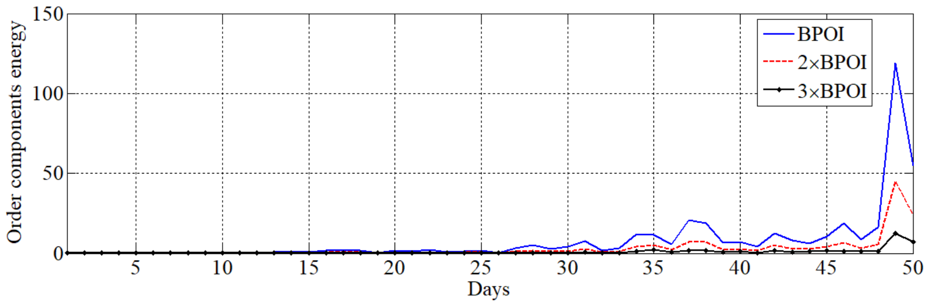
(a) FTO



(b) BSO



(c) BPOO



(d) BPOI

Figure 4 – Evolution of the order component energy for the different type of fault that can occur on the bearing over the 50 days of measurement: (a) cage fault, (b) Rolling elements fault, (c) Outer-race fault and (d) Inner-race fault.

the 50 days. Concerning the order components located at $i \times$ FTO and $i \times$ BSO with $i = 1,2,3$, their energy remains constant till the 49th day where an amplification is observed. This may be related to a degradation of the bearing train and rolling elements at the end of the test. About the BPOO components, their energy is almost constant and negligible compare to that of the FTO and BSO. Therefore, the bearing outer-race is healthy. Otherwise, the energy of the order components located at $i \times$ BPOI increases along the days. This amplification is related to the evolution of the inner-race fault severity. A significant jump in the energy can be seen from the 30th day. According to the inspection done after the 50th day, a crack in the inner-race has been noticed at the 50th day. Thus, the proposed order tracking approach is efficient to detect earlier a bearing fault. Based on the bearing order component estimation provided by the H_∞ filter, different indicators can be designed to monitor bearing health state.

5 Conclusion

In this paper, an order tracking technique was proposed to diagnose a bearing fault under a nonstationary condition. The proposed method consists of estimating a certain number of bearing order components whitout removing the deterministic components. The method described the squared envelope signal in the state space model using a trigonometric series expansion. Then, an H_∞ filter is designed to track bearing fault order components. Firstly, the theoretical foundation of the proposed approach has been described in details. Secondly, a synthetic signal has been generated to evaluate the performance of the proposed approach. It has been shown that the approach was able to track the bearing order components without removing the deterministic components. Moreover, the performance of the proposed approach remains stable for an uncertainty error on bearing orders less than 2.5%. Finally, the efficiency of the proposed approach has been demonstrated with wind turbine vibration signals under a nonstationary condition. The order components related to the bearing fault has been estimated by the proposed approach throughout 50-days of measurement. A fault on the bearing inner-race has been successfully detected earlier at the 30th day. In terms of perspective for this research, the authors will work on the design of a robust H_∞ filter to deal with a large uncertainty on the order of the bearing fault.

Acknowledgements

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A Synthetic meshing signal generation

The meshing signal is composed of five components and presented as follows:

$$y_g[k] = \kappa(\omega[k]) \sum_{m=1}^{5} y_{g,m}[k] \tag{35}$$

where $y_{g,m}$ is the mth meshing component. The latter is defined by the expression below:

$$y_{g,m}[k] = A_m[k](1 + a_m[k])e^{o_m \theta[k]} \tag{36}$$

in which:

- a_m is the amplitude modulation of the mth meshing component;
- A_m is the amplitude of the mth meshing component so that it is a random value comprises between 0 and 1 and $\kappa(\omega[k]) = \omega^2[k]/\max(\omega[:])$ is the modulation correponding to the variation of the regime in which ω is the angular speed of the machine;
- $\theta[k] = 2\pi t_s \sum_{c=1}^{k} f_r[c]$ is the instantaneous angular displacement of the shaft rotating at frequency f_r. t_s is the sampling period;

- o_m corresponds to the mth order of the meshing signal. It takes respectively the following values:45, 2×45, 3×45, 1.089×49 and $2\times 1.089\times 49$.

It is well known that when a fault appears on a gear tooth, the resonance frequency of the gear structure is excited by an impulse [6]. The generated impulse signal modify the shape of the amplitude modulation so that the latter can be described by a series of impulsive. Each impulse is modeled by a narrow bandwidth gausian function. This function is defined by as below:

$$ext[k] = \sum_i e^{-\frac{1}{2}\left(\frac{(k-1)t_s-\mu_i}{\sigma^2}\right)^2} \quad (37)$$

where μ_i is the center of the Gaussian function determined by the time instant for which the impulse occurs on the gear tooth and σ defines the width of the Gaussian function. In this simulation $\sigma = 10^{-3}$. Since the gear attenuates the impulse generated by the fault, a transfert function is included in the model so that the amplitude modulation becomes:

$$a_m[k] = \rho_m \times s[k] \circledast ext[k] \quad (38)$$

in which $s[k] = e^{-\xi^{(g)}(k-1)t_s} \sin\left(2\pi f_{res}^{(g)}(k-1)t_s\right)$ is the transfert function of the gear structure and ρ_m is a random value comprises between 0 and 1. $\xi^{(g)}$ and $f_{res}^{(g)}$ are respectively the damping factor and the resonance frequency of the gear structure. They are respectively equal to 5000 and 4000 Hz.

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