Synchronous fitting for deterministic signal extraction in non-stationary regimes: Application to helicopter vibrations

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Abstract

Deterministic-random separation is crucial in machine signal processing. The synchronous average is a widely used tool that separates the deterministic contribution from the random one. This tool consists on averaging the cycles of the vibration signal. In fact, it uses the fact that, for a given location in the cycle, the associated samples have a constant mean. This makes possible to estimate the signal mean through synchronous averaging, i.e. by averaging the samples associated with each position in the cycle. However, in many practical applications, the cycle-to-cycle statistics can change according to many factors such as the speed, torque, load, etc. The resulting signal is widely referred in the literature as cyclo-non-stationary. This means that the mean signal is not periodic anymore, thus jeopardizing the synchronous average technique. This paper addresses this issue by proposing a new generalization of the synchronous average. The proposed method takes advantage of the smoothness of the statistics (in particular the mean) variation with respect to cycles. Instead of computing the average of the samples located at a given angular location, the time-varying mean is computed by optimally fitting the data with an appropriate curve. This defines the synchronous fitting idea, being a mean estimator of cyclo-non-stationary signals. Two solutions are proposed to solve the fitting problem. Whereas the first seeks for a global solution, the second adopts a local solution inspired from Savitzky–Golay filter. These two approaches are tested and compared on numerical and real signals captured from a helicopter engine operating under a runup regime. Overall, the results have asserted the superiority of the local approach over the global one.

1. Introduction

The theory of cyclostationary processes has proven to be effective in describing and processing rotating machine signals [Antoni 2009]. The vibration components generated by mechanical sources can be mainly classified into first and second (or higher) order cyclostationary classes. First order cyclostationary components are those deterministic, principally consisting of a set of sinusoids corrupted with stationary noises. Those are described by the (quasi-) periodicity of their mean. Examples of first-order phenomena can include gear meshing vibrations, shaft unbalance and misalignment, fan rotations and others. Second-order cyclostationary components are random in nature, meaning that their mean equals zero or, equivalently, their spectrum does not exhibit clean harmonics. The periodicity of these components is hidden and can be revealed through the instantaneous power or, more generally, the auto-covariance function. In practice, those components are generated by different kind of mechanical phenomena subjected to some randomness. A typical example is the vibrations generated by a local fault in a rolling element bearing wherein the randomness is due to the presence of a slippage in the motion of the rolling elements [Ho 2000]. For this reason, the cyclostationary analysis offers an efficient way to detect and characterize the presence of clear or hidden periodicities in the signals through a thorough differential diagnosis. Obviously, the separation of first and second-order components is crucial for an accurate analysis of the signal. Nowadays, the corresponding state of the art comprises a set of supervised and unsupervised signal processing tools that deal with this issue. Among these methods, one of the most widely used is the synchronous average (SA) [Braun 1975]. The latter simply consists of cutting the signal into slices of the same length, being equal to the fundamental period of the extracted component and averaging them together. As it will be shown later, this paper deals with its generalization.

The cyclostationary modelling assumes the (hidden-) periodicity to be stable in time, which in turn requires a constant speed. Such a condition is however hard to obtain as the speed often undergoes some fluctuations. This jeopardizes the effectiveness of the SA even if the magnitude of the speed fluctuations is low. Since repetitive patterns in rotating machines are intrinsically locked to specific angular positions, it totally makes sense to rather process the signal in the angular domain. In this case, the cyclostationary property holds in the angle domain and, consequently, the SA is applied on the angular signal, either obtained by angular sampling or resampling [Antoni 2004].

In the case of large speed fluctuations, signals are subjected to significant distortions that jeopardize the effectiveness of the SA. These distortions are basically introduced by (i) variations of the machine power intake and (ii) the effect of linear time-invariant (LTI) transfers. Whereas the former essentially results in amplitude modulation, the latter also induces phase modulation. Non-periodic modulations obviously invalidate the (angle-) CS assumption and call for a more general description of nonstationary signals. Accordingly, the principle of cyclo-non-stationarity was proposed to formalize this specific type of signals. The consideration of cyclo-non-stationary signals requires the extension of the cyclostationary signals. This paper is particularly concerned in extending the synchronous average. Many previous works have addressed this issue. Reference [Coats 2009] proposed the improved synchronous average, being based on resampling the signal with a virtual tachometer signal synthesized via the demodulated phase. Another attempt to generalize the SA was proposed in Ref. [Daher 2010] through a parametric approach. In details, the authors used the Hilbert space representation of the deterministic component in which they decomposed the deterministic components onto a set of periodic functions multiplied by speed-dependent functions apt to capture long-term evolution over consecutive cycles. In ref. [Abboud 2016], the authors proposed a non-parametric approach based on averaging the signal cycles that belong to a given regime, defined by its central speed and a pre-defined width.

This paper proposes a different approach to generalize the SA based on a synchronous curve fitting of the data. The theoretical backgrounds of the proposed technique is exposed in section 2, while its performances are evaluated through numerical simulations in section 3. In section 4, the efficiency of the technique is tested on real vibration signals recorded under a varying speed condition.

2. Description of the synchronous fitting technique

In this section, the fundamentals of the proposed method are provided. First, a mathematical model for CNS signals is reviewed. Then, a global solution for the first order CNS estimation, which corresponds to the one proposed in [Daher 2009], is presented. Finally, the newly proposed technique based on a local solution is introduced.

2.1. General

Let $x[n]$ be a first-order CNS signal with a characteristic period N (i.e. cycle and $1/N$ the normalized frequency) and a length L. One can model such a signal as follows:

$$\forall n \in \{1, \dots, L\} \quad x[n] = d[n] + w[n] = \sum_k d_k[n] e^{\frac{j2\pi kn}{N}} + w[n] \tag{1}$$

where k is an integer, $d_k[n] \in \mathbb{C}$ are deterministic smooth functions (whose real and imaginary part are continuous and differentiable) and whose bandwidths, noted B_k, are much smaller than the half the fundamental frequency i.e. : $\forall k, B_k \ll 1/2N$ and $w[n]$ is a random noise. The discrete-time Fourier transform (DTFT) of (1) reads:

$$\forall f \in]-1/2; 1/2] \quad X[f] = \sum_k D_k[f] * \delta(f - k/N) + W[f] \tag{2}$$

where $D_k[f]$ and $W[f]$ are respectively the DTFTs of $d_k[n]$ and $w[n]$. Since $d_k[n]$ are deterministic smooth functions, and according to the Weirstrass theorem, they can be approximated through a P-order polynomial function, i.e.:

$$\forall k \quad d_k[n] \approx \sum_{p=0}^{P} d_k^p n^p \tag{3}$$

where $d_k^p \in \mathbb{C}$. By inserting Eq. (3) into the expression of $d[n]$, one obtains:

$$\forall n \in \{1, \dots, L\} \quad d[n] = \sum_{p=0}^{P} c_p[n] n^p \tag{4}$$

where $c_p(n) = \sum_k d_k^p e^{j2\pi kn/N}$ is a periodic function of period N. Equation (4) indicates that the deterministic component can be approximated by a sum of periodic functions multiplied with the polynomial basis: it is actually a polynomial with periodic coefficients.

Let's first define $\bar{n} = \lfloor (n-1)/N \rfloor + 1$ as the sample location within the period N ($\lfloor a/b \rfloor$ denotes the remainder of the division of a by b) . Since $c_p(n)$ is periodic with period N, we have $c_p[\bar{n}] = c_p[\bar{n} + (q-1)N]$ for all integer $q = 1, \dots, Q$ (Q is the number of cycles). Thus, Eq. (4) can be equivalently written as follows:

$$\forall q \in \{1, \dots, Q\} \ \forall \bar{n} \in \{1, \dots, N\} \quad d[\bar{n} + (q-1)N] = \sum_{p=0}^{P} c_p[\bar{n}](\bar{n} + (q-1)N)^p \tag{5}$$

Using the binomial theorem ($(\bar{n} + (q-1).N)^p = \sum_{i=0}^{p} C_i^p N^i \, (\bar{n} - N)^{p-i} q^i$ where C_i^p is the binomial coefficient), one can deduce from Eq. (5) that the samples associated with the same location $\bar{n}$ in the period, $s_q[\bar{n}] = d[\bar{n} + (q-1).N]$ for all integer $q \in \{1, \dots, Q\}$, defines a polynomial of order P with constant coefficient, i.e.:

$$\forall q \in \{1, \dots, Q\} \ \forall \bar{n} \in \{1, \dots, N\} \quad s_q[\bar{n}] = \sum_{p=0}^{P} b_p[\bar{n}] q^p \tag{6}$$

where $b_p[\bar{n}] = N^p \sum_{j=p}^{P} C_p^j \, (\bar{n} - N)^{j-p} c_j[\bar{n}]$. Note that $b_p[\bar{n}]$ is parametrized by $\bar{n}$.

2.2. A global LMS solution

In the case of a noisy signal $x[n]$, a good estimate of the deterministic component $d[n]$ is then to find the best fit of the curve $\boldsymbol{s}[\bar{n}] = \left[s_1[\bar{n}], \dots, s_Q[\bar{n}]\right]^T$ for each $\bar{n} \in \{1, \dots, N\}$ which reduces to find an estimate of $\boldsymbol{b}[\bar{n}] = \left[b_1[\bar{n}], \dots, b_P[\bar{n}]\right]^T$ for each $\bar{n} \in \{1, \dots, N\}$ for a given polynomial of order P. A common way to do this is to find the curve which minimizes the least mean square error, i.e.:

$$\begin{aligned} \forall \bar{n} \in \{1, \dots, N\} \quad \hat{\boldsymbol{b}}[\bar{n}] &= \operatorname{argmin}\left(\sum_{q=1}^{Q} w[\bar{n} + (q-1)N]^2\right) \\ &= \operatorname{argmin}\left(\sum_{q=1}^{Q}\left(d[\bar{n} + (q-1)N] - x_q[\bar{n}]\right)^2\right) \\ &= \operatorname{argmin}\left(\sum_{q=1}^{Q}\left(\sum_{p=0}^{P} b_p[\bar{n}] q^p - x_q[\bar{n}]\right)^2\right) \end{aligned} \tag{7}$$

where $x_q[\bar{n}] = x[\bar{n} + (q-1)N]$. Let's define the $Q \times (P+1)$ matrix $\boldsymbol{\Phi}$ such that $\boldsymbol{\Phi}_{q,p} = q^{p-1}$ (with $q \in \{1, \dots, Q\}$ and $p \in \{1, \dots, P+1\}$, and $\boldsymbol{x}[\bar{n}] = \left[x_1[\bar{n}], \dots, x_Q[\bar{n}]\right]^T$. One can rewrite the Eq. (7) as:

$$\forall \bar{n} \in \{1, \dots, N\} \ \hat{\boldsymbol{b}}[\bar{n}] = \operatorname{argmin}||\boldsymbol{\Phi}\boldsymbol{b}[\bar{n}] - \boldsymbol{x}[\bar{n}]||^2 \tag{8}$$

whose solution expresses as follows:

$$\forall \bar{n} \in \{1, \dots, N\} \ \hat{\boldsymbol{b}}[\bar{n}] = \left(\boldsymbol{\Phi}^{\mathbf{T}}\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^{\mathbf{T}} \, \boldsymbol{x}[\bar{n}] \tag{9}$$

Once the coefficients are calculated, one can find the estimated deterministic component located at $\bar{n}$ in the form:

$$\forall \bar{n} \in \{1, \dots, N\} \;\; \hat{\boldsymbol{s}}[\bar{n}] = \boldsymbol{\Phi}\hat{\boldsymbol{b}}[\bar{n}] = \boldsymbol{\Phi}\left(\boldsymbol{\Phi}^{\mathbf{T}}\boldsymbol{\Phi}\right)^{-1}\boldsymbol{\Phi}^{\mathbf{T}}\,\boldsymbol{x}[\bar{n}] \tag{10}$$

The deterministic signal can then be deduced as follows:

$$\forall n \in \{1, \dots, L\} \;\; d[n] = s_q[\bar{n}] \;\; \text{where} \;\; \bar{n} = \lfloor (n-1)/N \rfloor + 1 \;\; \text{and} \;\; q = 1 + (n - \bar{n})/N \tag{11}$$

2.3. A local LMS solution

This subsection describes the proposed method. The basic idea is excerpted from the "Savitzky-Golay filter" which is a widely known method to smooth or fit the data based on the least mean square solution of local polynomial fitting [Savitzky 1964]. Precisely, for every $q \in \{1, \dots, Q\}$, let's consider the data set $x_q[\bar{n}]$ being a function of q and parametrized by $\bar{n}$; we try to find the best LMS polynomial fit, with a fixed order P at the point $\bar{n}$, from the $2M+1$ subset centered at q, i.e. $\{x_{q-M}[\bar{n}], \dots, x_{q+M}[\bar{n}]\}$. That being said, this problem can be stated in a similar way as the previous subsection, i.e.:

$$\forall\, q \in \{1, \dots, Q\}\, \forall \bar{n} \in \{1, \dots, N\} \;\;\; \hat{\boldsymbol{b}}^{(q)}[\bar{n}] = \operatorname{argmin}\left\|\mathbf{J}\,\boldsymbol{b}^{(q)}[\bar{n}] - \boldsymbol{x}^{(q)}[\bar{n}]\right\|^2 \tag{12}$$

where $\boldsymbol{x}^{(q)}[\bar{n}] = \left[x_{q-M}[\bar{n}], \dots, x_{q-M}[\bar{n}]\right]^{\mathsf{T}}$ represents the q^{th} subset, $\boldsymbol{b}^{(q)}[\bar{n}] = \left[b_0^{(q)}[\bar{n}], \dots b_P^{(q)}[\bar{n}]\right]^{\mathsf{T}}$ are the $P+1$ polynomial coefficients associated with the q^{th} subset, and $\mathbf{J}$ the $(2M+1) \times (p+1)$ matrix such that $\forall m \in \{1, \dots, 2M+1\}\, \forall p \in \{1, \dots, P+1\}$ $\mathbf{J}_{m,p} = (m - M + 1)^{p-1}$. The $(2M+1)$-length curve that best fits the q^{th} subset writes:

$$s_m^{(q)}[\bar{n}] = \sum_{p=0}^{P} b_p^{(q)}[\bar{n}]\,.\,(m - M + 1)^p \tag{13}$$

The Savitzky-Golay method suggests to estimate the deterministic component at the q^{th} data point by retaining the value of the polynomial at the central point i.e. at $m = M + 1$:

$$\forall\, q \in \{1, \dots, Q\}\, \forall \bar{n} \in \{1, \dots, N\} \;\; \hat{d}[\bar{n} + (q-1).N] = \hat{s}_{M+1}^{(q)}[\bar{n}] = b_{M+1}^{(q)}[\bar{n}] \tag{14}$$

Following the same lines as for Eq (9), one can show that the coefficients of the polynomial write:

$$\forall\, q \in \{1, \dots, Q\}\, \forall \bar{n} \in \{1, \dots, N\} \;\;\; \hat{\mathbf{b}}^{(q)}[\bar{n}] = \boldsymbol{H}\,\boldsymbol{x}^{(q)}[\bar{n}] \tag{15}$$

with $\boldsymbol{H} = \left(\mathbf{J}^{\mathbf{T}}\mathbf{J}\right)^{-1}\mathbf{J}^{\mathbf{T}}$ a matrix of size $(P+1) \times (2M+1)$ whose elements are independent of $\bar{n}$ and q . The $(M+1)^{th}$ element of the above vector namely $\hat{b}_{M+1}^{(q)}[\bar{n}]$ is actually a linear combination of $\boldsymbol{x}^{(q)}[\bar{n}]$ with the $2M+1$ elements of the $(M+1)^{th}$ row, $\boldsymbol{h}^{\boldsymbol{T}} = [h_{-M}, \dots h_M]$, of $\boldsymbol{H}$ being independent of q and $\bar{n}$:

$$\hat{b}_{M+1}^{(q)}[\bar{n}] = \boldsymbol{h}^{\boldsymbol{T}}\boldsymbol{x}^{(q)}[\bar{n}] \tag{16}$$

Considering equations (14) and (15), one can write the estimate of the deterministic component

$$\begin{aligned} \hat{d}[\bar{n} + (q-1).N] &= \sum_{m=-M}^{M} x_{q-m}[\bar{n}]\, h_m \\ &= \sum_{m=-M}^{M} x[\bar{n} + (q-1).N - m.N]\, h_m \\ &= \sum_{i=-MN}^{MN} x[\bar{n} + (q-1).N - i]\, \tilde{h}_i \end{aligned} \tag{17}$$

where $\tilde{\boldsymbol{h}}^{\boldsymbol{T}} = [\tilde{h}_{-MN}, \dots, \tilde{h}_{MN}\ ]$ is obtained by zero-padding $\boldsymbol{h}$ as follows:

$$\begin{cases} \tilde{h}_i = h_m & if\ \ i = mN,\ \forall - M \leq m \leq M \\ \tilde{h}_i = 0 & elsewhere \end{cases} \tag{18}$$

It becomes obvious that the estimated deterministic component turns to a LTI filtering of the original signal $x[n]$ with the $(2MN + 1)$-length filter $\tilde{\boldsymbol{h}}_i$:

$$\hat{d}[n] = \sum_{i=-MN}^{MN} x[n-i]\, \tilde{h}_i \tag{19}$$

3. Numerical evaluation

In this section, the performance of the synchronous fitting techniques are tested and compared on a synthetic signal. The deterministic signal is modelled as a sum of four speed-varying sinusoids whose envelopes and phases are functions of the cyclo-non-stationary $\lambda[n]$ (which can be in practice the torque, load, speed, etc.):

$$d[n] = \sum_{k=1}^{4} A_k[n] \sin(2\pi kn/N + \Phi_k[n]) \tag{20}$$

Where:

- $A_k[n]$ and $\Phi_k[n]$ are functions of $\lambda[n]$ (see Fig. 1);
- $N = 100$ is the fundamental period;
- $L = 15000$ is the signal length.

The deterministic signal is exposed in Fig 2 together with its noisy version constituted by adding a white Gaussian noise such that the initial signal to noise ratio is equal to -3 dB.

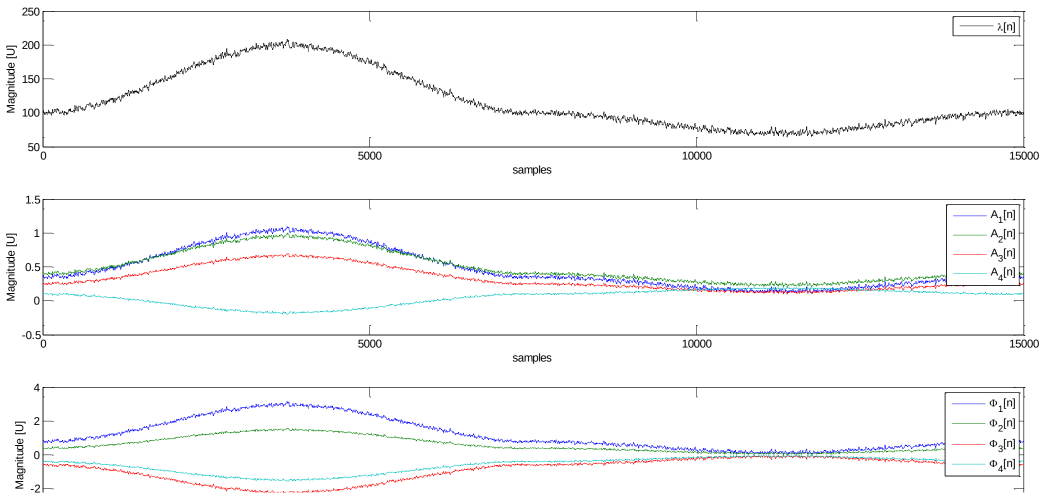

Figure 1: The plot of the cyclo-non-stationary agent $\lambda[n]$ (top), the 4 amplitudes $A_k[n]$ (middle) and the 4 phase modulations $\Phi_k[n]$ (bottom) associated with the sinusoids of the synthetic signal.

In the following, the global and local approaches are applied to the noisy signal with respect to the cycle N with the aim of recovering the deterministic component, being here the signal of interest. For the global approach, the degree of polynomial was set to 30, knowing that the results were stable for higher polynomial degrees. For the local approach, the window length was set to 49 (i.e. $M = 24$) and the polynomial order to 3. The obtained results are exposed in Fig. 3 together with the error signal obtained by subtracting the estimated signal from the actual one. Both approaches tend to estimate with good accuracy the deterministic signal, with a clear superiority of the newly proposed local approach over the global one: the estimation error of the latter is almost twice larger than the former.

Eventually, the performance of these methods are compared for different signal-to-noise ratios (SNR). For this purpose, the relative error defined as the energy of the error normalized by the signal energy:

$$\epsilon_n[n] = 10\log_{10}\left(\sum_{n=1}^{N}\left(d[n]-\hat{d}[n]\right)^2 / \sum_{n=1}^{N} d[n]^2\right). \tag{21}$$

The obtained results are exposed in Fig. 4. The SA returns poor and consistent results as this latter only estimates the average periodic component existing in the signal. The reason is that the number of average is big so it was slightly affected by the SNR: the average periodic part was almost the same for all SNR. When it comes to the synchronous fitting techniques, the local approach evidences better estimation performances. In fact, the local approach returns an estimation error less than -6 dB when the SNR greater than -6 dB, while the global approach needs a SNR greater than 10 dB to get this accuracy. The results highlight the effectiveness of the local approach as compared with the global one. The reason is that the local approach assumes that the mean is locally smooth which is a more accurate assumption.

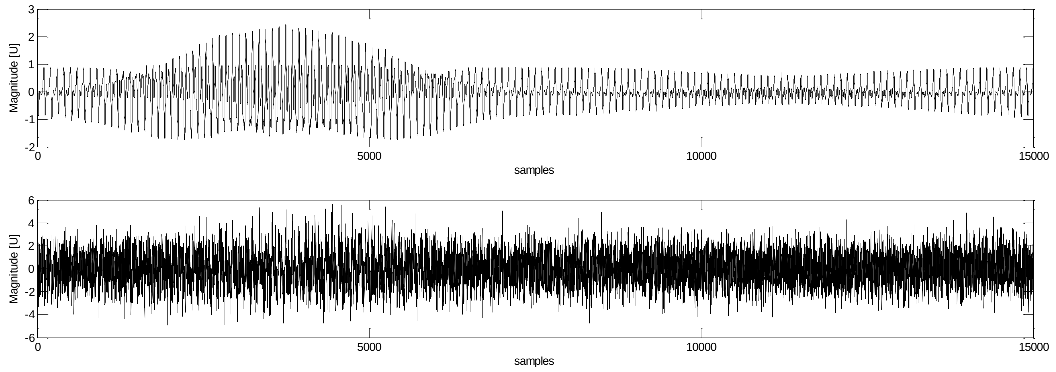

Figure 2: The deterministic component (top plot) and the noisy signal (bottom plot) constituted by adding a white Gaussian noise whose standard deviation equals twice of the former.

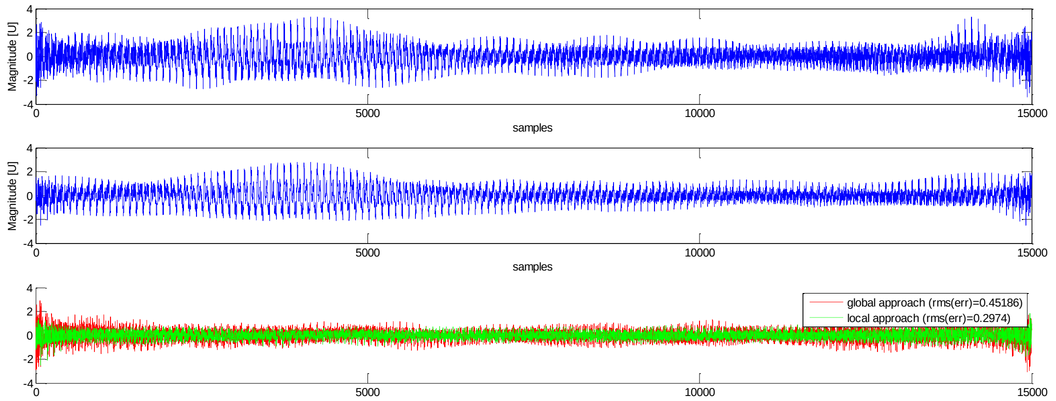

Figure 3: The deterministic component (top plot) and the noisy signal (bottom plot) constituted by adding a white Gaussian noise whose standard deviation equals twice of the former.

4. Application: a helicopter engine

In this section, the proposed approach is applied on real vibration signals captured from the gas generator of a helicopter engine. The aim is to extract the component related to the centrifugal compressor while the engine speed operates under a runup regime. An encoder is also present to measure the shaft location and to provide an accurate estimation of the engine speed. The encoder signal is used to resample the signal in the angular domain and the synchronous fitting techniques are both applied with respect to the blade pass period of the centrifugal compressor. It is worth noting that the blade pass period equals the shaft period divided by the compressor blade number. The blade number as well as the signals magnitude are not given for confidentiality reason. Figure 5 exposes the raw acceleration signal, the synchronous fitting estimations via the global and local approach. It is obvious that the signal associated with the local approach is much more accurate presenting a clear resonance starting at 10s. The related spectrograms are exposed in Figure 6 wherein the speed-varying harmonics of the centrifugal compressor are clearly shown. Though the extraction seems good in both techniques, it was hard to compare the performance of the extraction techniques. The order spectrum of the signals are computed by the Welch estimator applied to the angular resampled signals. The obtained spectra are exposed in Fig. 6 for comparison. Whereas the latter show better noise rejection in the global solution case especially for the noise floor, the close-ups clearly evidence the superiority of the local approach in accurately estimating the peaks. The global approach tends to loose accuracy as the frequency (order) gets high, this is because the global interpolation tends to confuse high frequency sinusoids with high frequency noises. Overall, the newly proposed local approach evidences better performances than the global one.

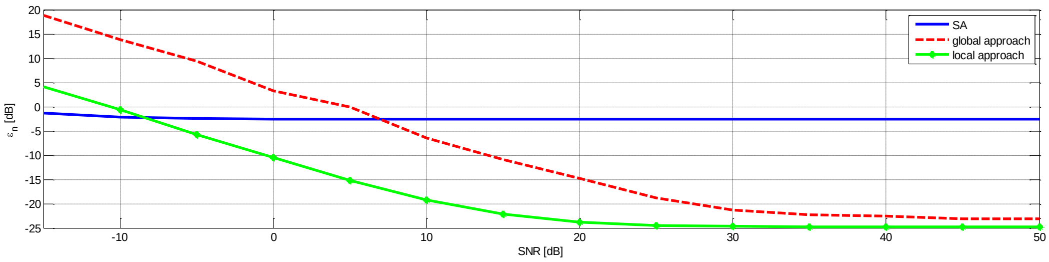

Figure 4: Performance of the SA, the synchronous fitting with the global and the local approach.

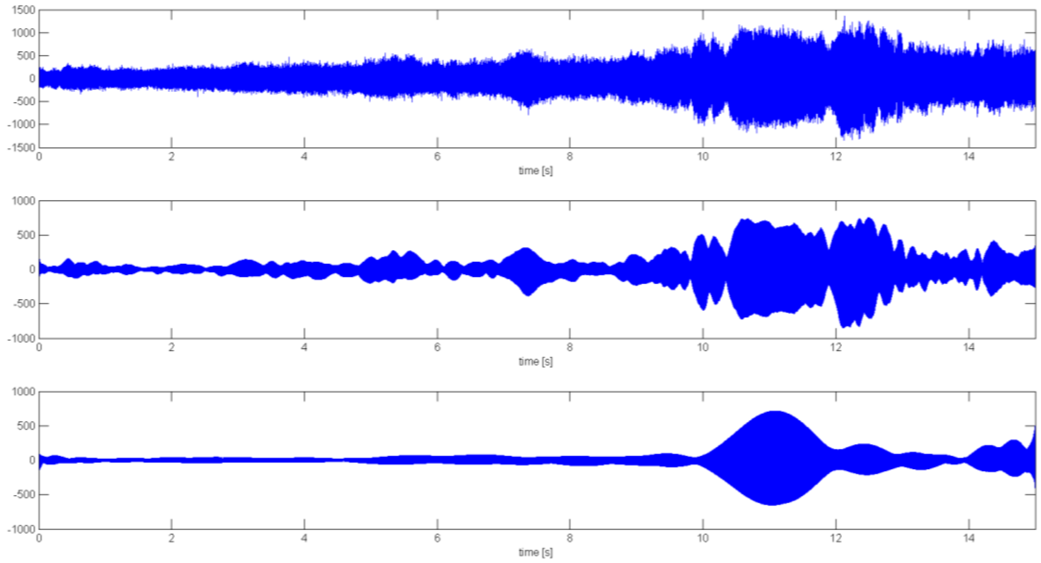

Figure 5: The raw acceleration signal (top), the synchronous fitting with the global approach (middle) and local approach (bottom).

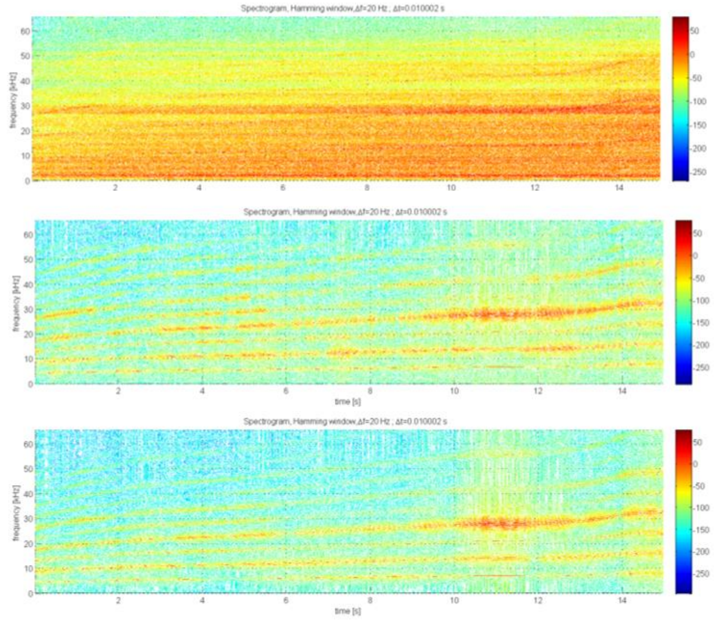

Figure 6: Spectrograms of the raw acceleration signal (top), the synchronous fitting with the global approach (middle) and local approach (bottom).

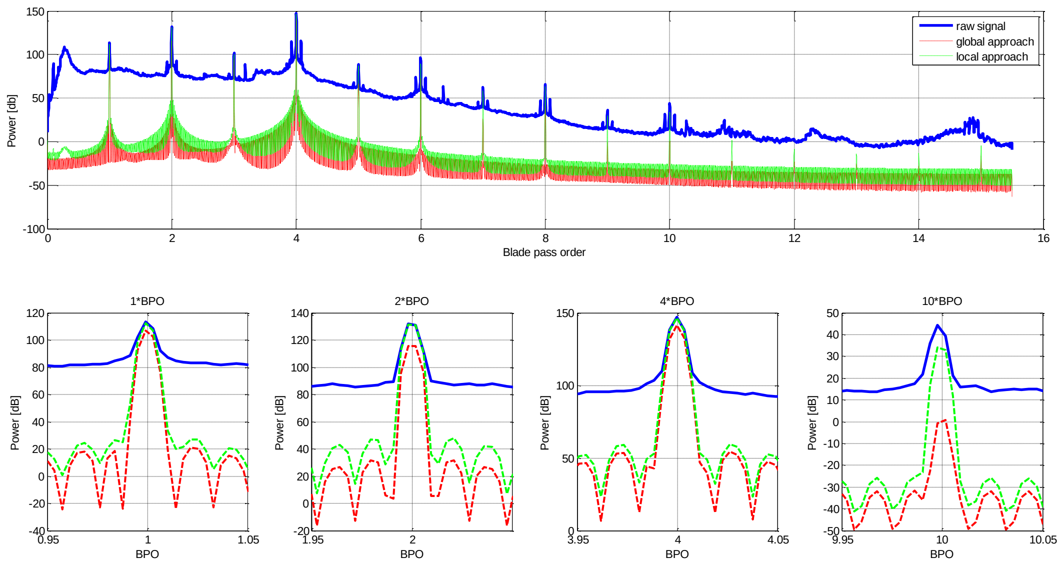

Figure 7: Order spectra of the raw acceleration signal (blue continuous line), the synchronous fitting with the global approach (red dotted line) and local approach (green dotted line).

5. Conclusion

This paper proposes a new technique for the extraction of a deterministic component in variable regimes. It can be seen as an extension of the classical synchronous averaging. In fact, instead of computing the cyclic mean via the synchronous average, the latter is computed via a synchronous fitting. This leads to two ways to tackle the issue. The first way seeks a global solution and compute the mean, for the data associated with a given position in the cycle, by finding the best polynomial that minimizes the least mean square error. It turns out that the solution of this problem was previously proposed in a previous publication. However, the second way is original and addresses differently the same problem by seeking a local solution based on the Savitzky-Golay filter. Numerical simulations are conducted showing a clear superiority of the newly proposed local approach over the global one. In fact, the local approach returns an estimation error less than -6 dB when the SNR greater than -6 dB, while the global approach needs an SNR greater than 10 dB to get this accuracy. The results highlights the effectiveness of the local approach as compared with the global one. An additional advantage of the local approach over the global one is the computational cost. As the first turns to a linear-time-invariant convolution, its implementation is much easier than solving a global least mean square problem which requires a matrix inversion. Eventually, both techniques are successfully tested on a real vibration signal measured on a helicopter engine under a runup condition with the aim of extracting the vibratory component emitted by the centrifugal compressor of the gas generator. Both techniques were able to extract the component of interest, yet the local approach evidences much better extraction than the global one.

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